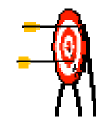
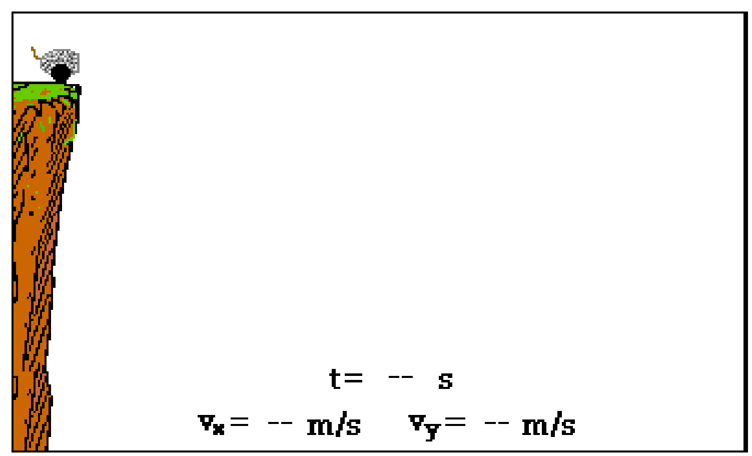
Projectile Motion

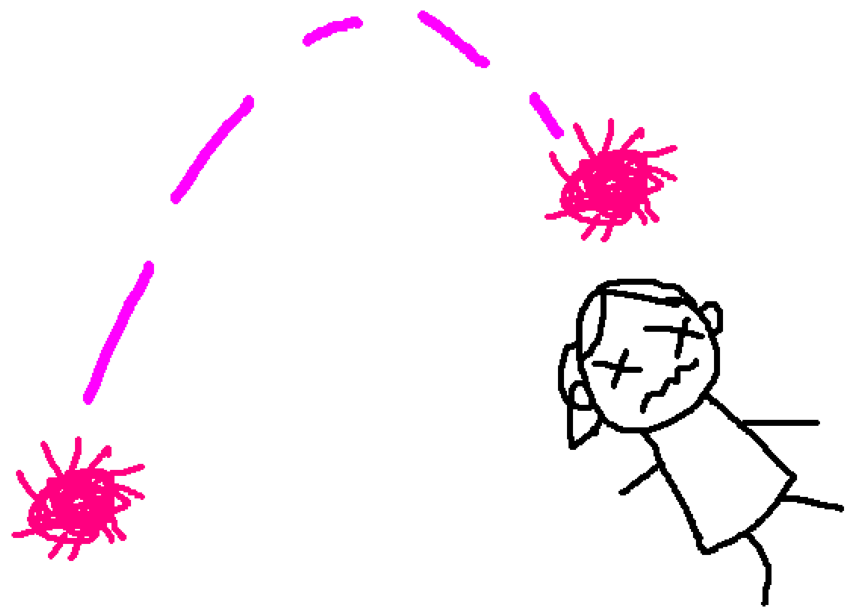


AP Physics C



What is projectile?

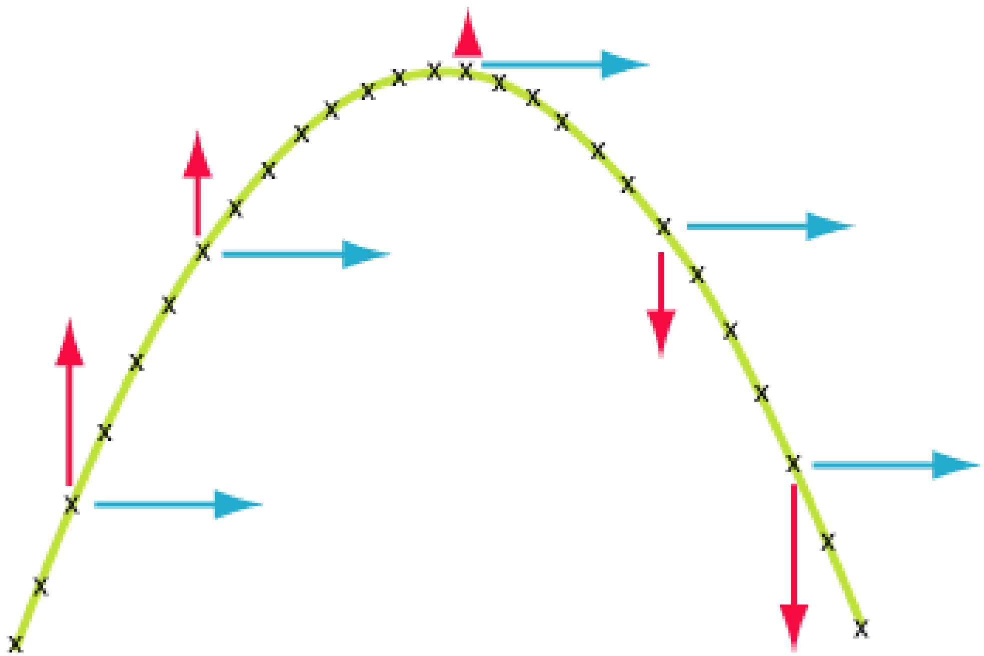
Projectile -Any object which projected by some means and continues to move due to its own inertia (mass).



Projectiles move in TWO dimensions

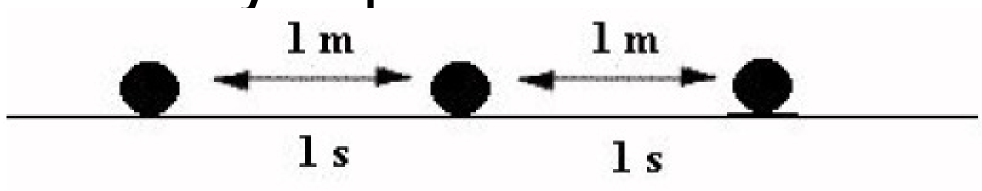
Since a projectile moves in 2-dimensions, it therefore has 2 components just like a resultant vector.

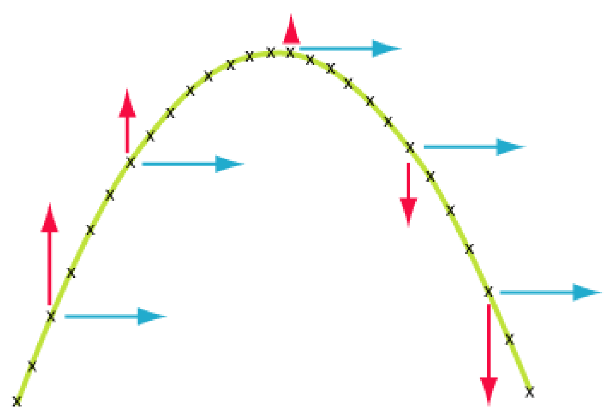
- **Horizontal and Vertical**



Horizontal "Velocity" Component

- **NEVER** changes, covers equal displacements in equal time periods. This means the initial horizontal velocity equals the final horizontal velocity



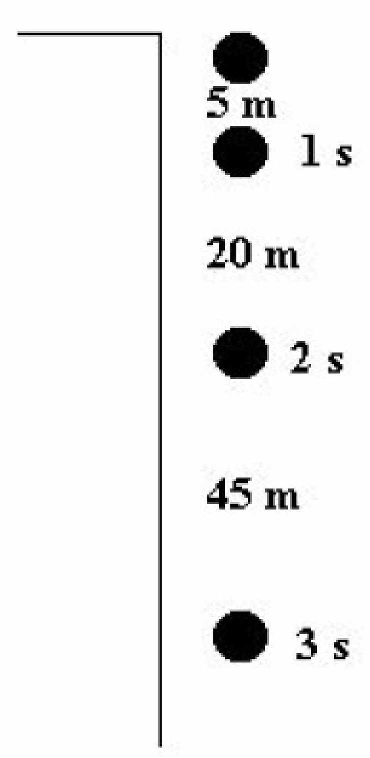


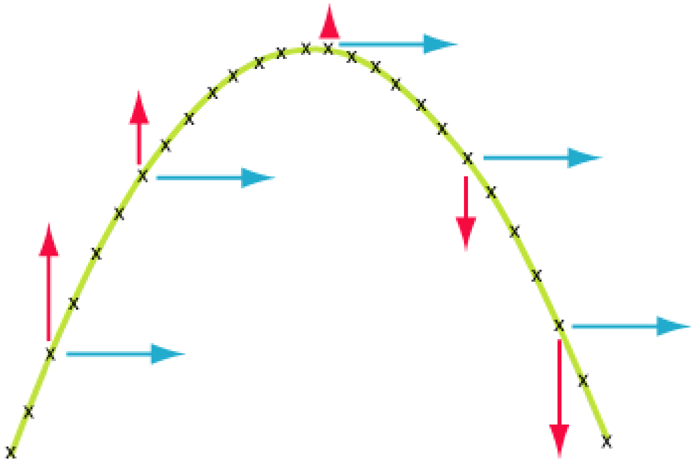
In other words, the horizontal velocity is CONSTANT. BUT WHY?

Gravity DOES NOT work horizontally to increase or decrease the velocity.

Vertical "Velocity" Component

- Changes (due to gravity), does **NOT** cover equal displacements in equal time periods.

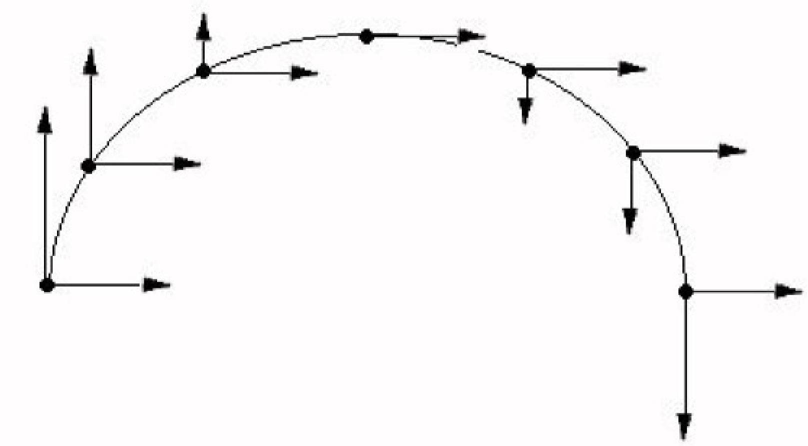




Both the MAGNITUDE and DIRECTION change. As the projectile moves up the MAGNITUDE DECREASES and its direction is UPWARD. As it moves down the MAGNITUDE INCREASES and the direction is DOWNWARD.

Combining the Components

Together, these components produce what is called a **trajectory** or path. This path is **parabolic** in nature.



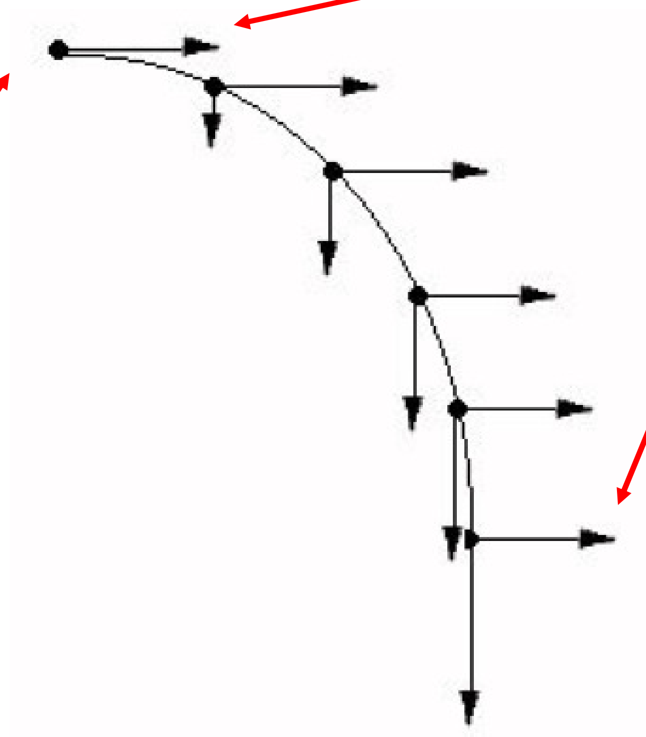
Component	Magnitude	Direction
Horizontal	Constant	Constant
Vertical	Changes	Changes

Horizontally Launched Projectiles

Projectiles which have NO upward trajectory and NO initial VERTICAL velocity.

$$v_{ox} = v_x = \text{constant}$$

$$v_{oy} = 0\, m/s$$



Horizontally Launched Projectiles

To analyze a projectile in 2 dimensions we need 2 equations. One for the "x" direction and one for the "y" direction. And for this we use kinematic #2.

$$x = v_{ox}t + \frac{1}{2}at^2$$

$x = v_{ox}t$	$y = \frac{1}{2}gt^2$

Remember, the velocity is CONSTANT horizontally, so that means the acceleration is ZERO!

Remember that since the projectile is launched horizontally, the INITIAL VERTICAL VELOCITY is equal to ZERO.

Horizontally Launched Projectiles

Example: **A plane traveling with a horizontal velocity of 100 m/s is 500 m above the ground. At some point the pilot decides to drop some supplies to designated target below. (a) How long is the drop in the air? (b) How far away from point where it was launched will it land?**

What do I know?	What I want to know?
v_{ox}=100 m/s	t = ?
y = 500 m	x = ?
v_{oy}= 0 m/s	
g = -9.8 m/s/s	

$$y = \tfrac{1}{2} g t^2 \rightarrow -500 = \tfrac{1}{2}(-9.8)t^2$$

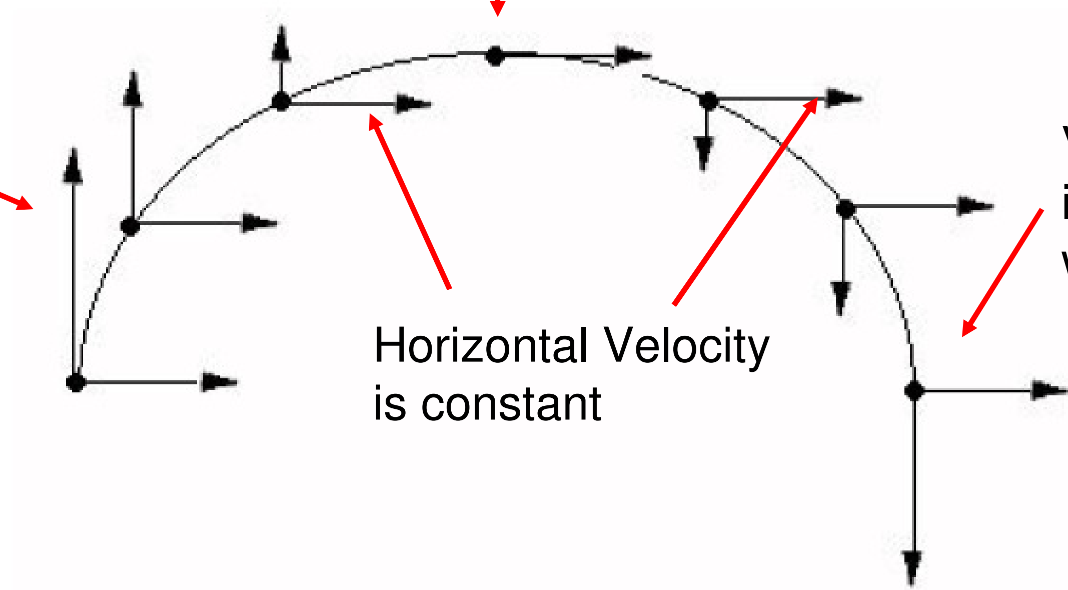
$$102.04 = t^2 \rightarrow t =$$ **10.1 seconds**

$$x = v_{ox} t = (100)(10.1) =$$ 1010 m

Vertically Launched Projectiles

NO Vertical Velocity at the top of the trajectory.

Vertical Velocity decreases on the way upward



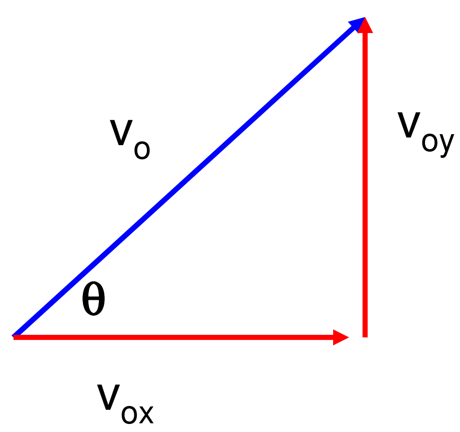
Horizontal Velocity is constant

Vertical Velocity increases on the way down,

Component	Magnitude	Direction
Horizontal	Constant	Constant
Vertical	Decreases up, 0 @ top, Increases down	Changes

Vertically Launched Projectiles

Since the projectile was launched at a angle, the velocity MUST be broken into components!!!

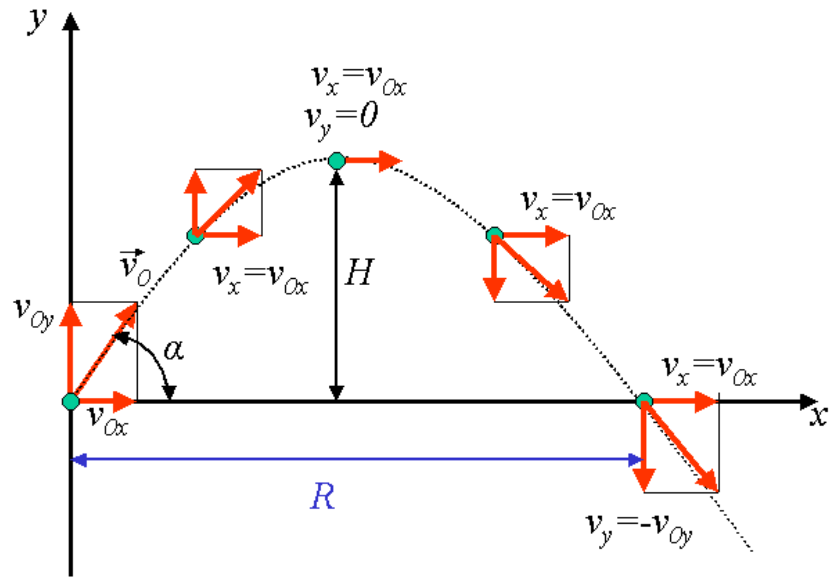


$$v_{ox} = v_o \cos\theta$$

$$v_{oy} = v_o \sin\theta$$

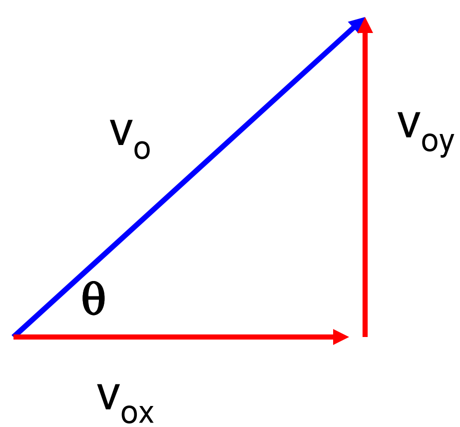
Vertically Launched Projectiles

There are several things you must consider when doing these types of projectiles besides using components. If it begins and ends at ground level, the "y" displacement is ZERO: y = 0



Vertically Launched Projectiles

You will still use kinematic #2, but YOU MUST use COMPONENTS in the equation.



$x = v_{ox} t$	$y = v_{oy} t + \frac{1}{2} g t^2$

$$v_{ox} = v_o \cos\theta$$

$$v_{oy} = v_o \sin\theta$$

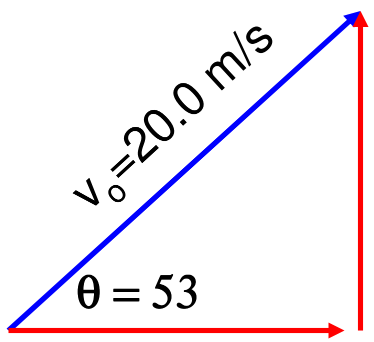
Example

A place kicker kicks a football with a velocity of 20.0 m/s and at an angle of 53 degrees.

(a) How long is the ball in the air?

(b) How far away does it land?

(c) How high does it travel?



$$v_{ox} = v_o \cos\theta$$

$$v_{ox} = 20\cos 53 = 12.04\ m/s$$

$$v_{oy} = v_o \sin\theta$$

$$v_{oy} = 20\sin 53 = 15.97\ m/s$$

Example

A place kicker kicks a football with a velocity of 20.0 m/s and at an angle of 53 degrees.

(a) How long is the ball in the air?

What I know	What I want to know
v_{ox}=12.04 m/s	t = ?
v_{oy}=15.97 m/s	x = ?
y = 0	y_{max}=?
g = - 9.8 m/s/s	

$$y = v_{oy}t + \tfrac{1}{2}gt^2 \rightarrow 0 = (15.97)t - 4.9t^2$$

$$-15.97t = -4.9t^2 \rightarrow 15.97 = 4.9t$$

$t =$ **3.26 s**

Example

A place kicker kicks a football with a velocity of 20.0 m/s and at an angle of 53 degrees.

(b) How far away does it land?

What I know	What I want to know
v_{ox}=12.04 m/s	t = 3.26 s
v_{oy}=15.97 m/s	x = ?
y = 0	y_{max}=?
g = - 9.8 m/s/s	

$$x = v_{ox}t \rightarrow (12.04)(3.26) =$$ **39.24 m**

Example

A place kicker kicks a football with a velocity of 20.0 m/s and at an angle of 53 degrees.

(c) How high does it travel?

CUT YOUR TIME IN HALF!

What I know	What I want to know
v_{ox}=12.04 m/s	t = 3.26 s
v_{oy}=15.97 m/s	x = 39.24 m
y = 0	y_{max}=?
g = - 9.8 m/s/s	

$$y = v_{oy}t + \frac{1}{2}gt^2$$

$$y = (15.97)(1.63) - 4.9(1.63)^2$$

$$y = \textbf{13.01 m}$$

A special case…

What if the projectile was launched from the ground at an angle and did not land at the same level height from where it started? **In other words, what if you have a situation where the "y-displacement" DOES NOT equal zero?**

Let's look at the second kinematic closely!

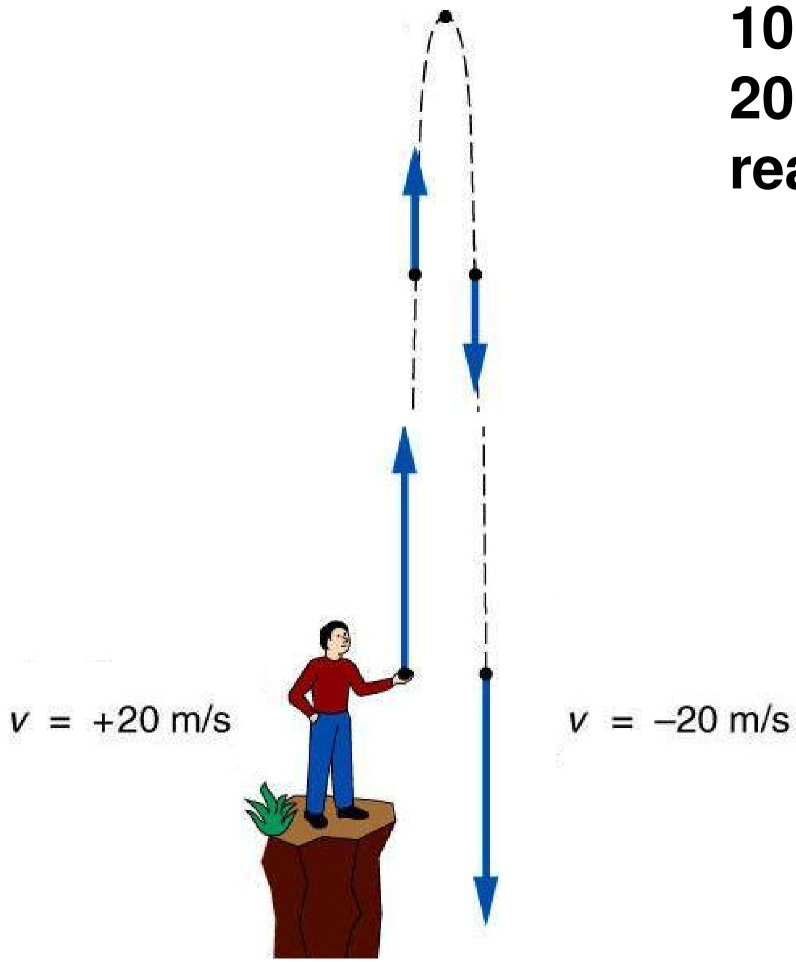
$$y = y_o + v_{o(y)}t + \frac{1}{2}gt^2$$

Assuming it is shot from the ground. We see we have one squared term variable, one regular term variable, and a constant number with no variable. What is this?

A QUADRATIC EQUATION!

A special case example

An object is thrown from the top of a 100m high cliff at an initial speed of 20 m/s. How long does it take to reach the ground?



Well, here is what you have to realize. As it goes up its speed decreases, reaches zero, the increasing back to the ground. When the ball reaches the cliff face it is now traveling at 20 m/s again , but in the opposite direction. DOWNWARD.

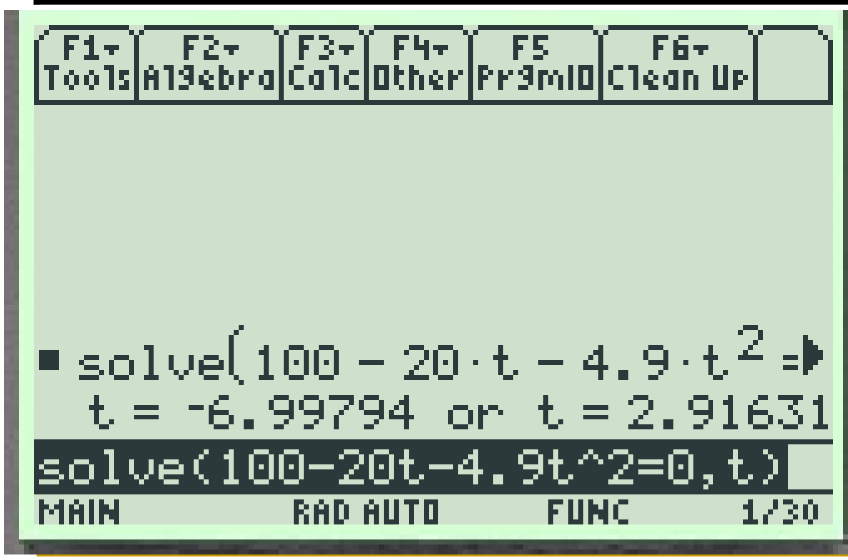
Thus the speed is considered to be -20 m/s.

Example

What I know	What I want to know
v_{oy}=-20.0 m/s	t = ?
y= 0 m	
y_0 = 100m	
g = - 9.8 m/s/s	

$$y = y_o + v_{oy}t + \frac{1}{2}gt^2$$

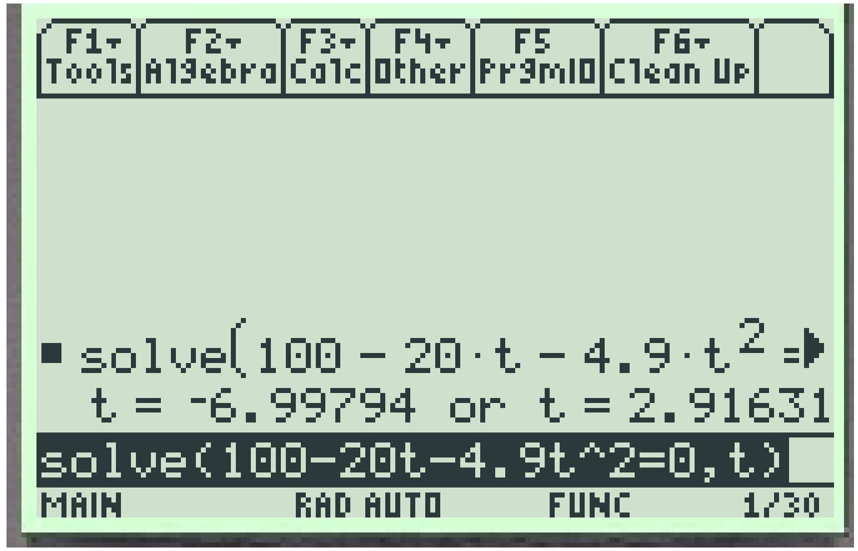
$$0 = 100 - 20t - 4.9t^2$$



GOTO F2 (which is the Algebra screen) **CHOOSE SOLVE**.

Type in the equation.
In this example our equation and set the it equal to zero. Then add a comma after the equation and tell it that you want it to solve for, in this case "t".

Example



It will respond by showing you exactly what you typed and the TWO ROOTS!

Since we are solving for time we choose the positive root as TIME cannot be negative. Thus it took 2.92 seconds for the ball to hit the ground.