Methods of Motion

AP Physics C

Displacement

Displacement (x or y) ***"Change in position"***

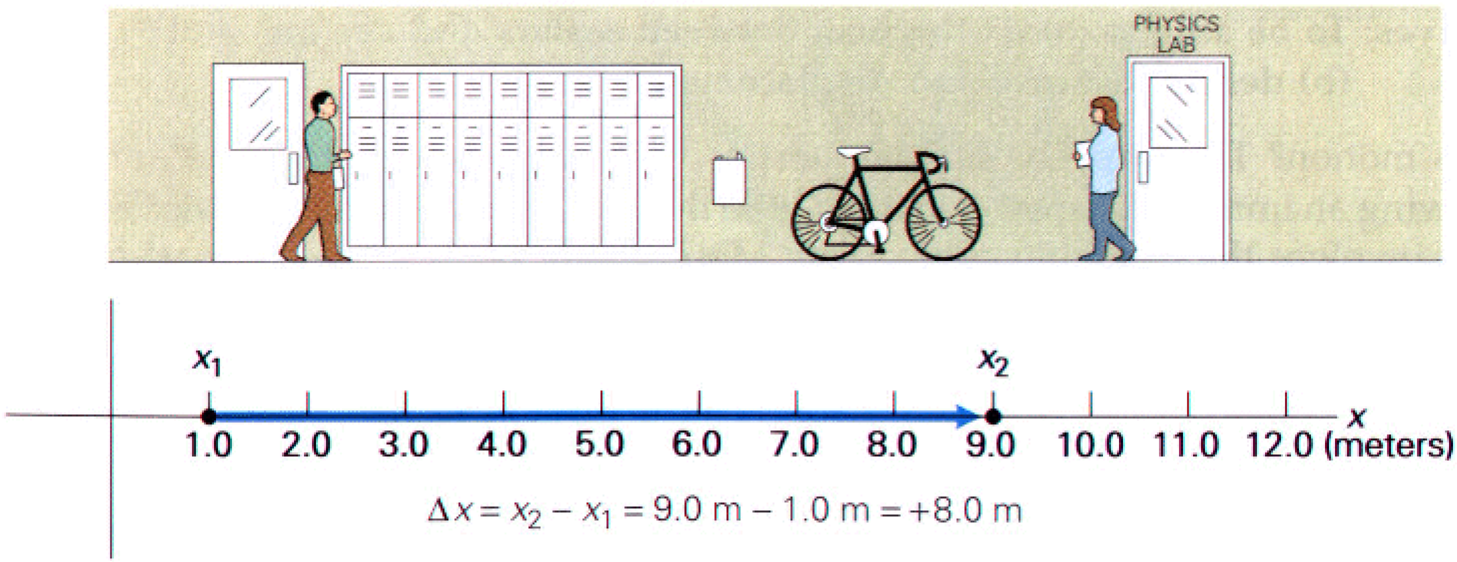
It is not necessarily the total distance traveled. In fact, displacement and distance are entirely different concepts. Displacement is relative to an axis.

- "x" displacement means you are moving horizontally either right or left.
- "y" displacement means you are moving vertically either up or down.
- The word ***change*** is expressed using the Greek letter **DELTA (Δ)**.
- To find the ***change*** you ALWAYS subtract your **FINAL - INITIAL** position
- It is therefore expressed as either $\Delta x = x_f - x_i$ or $\Delta y = y_f - y_i$

Distance - How far you travel regardless of direction.

Example

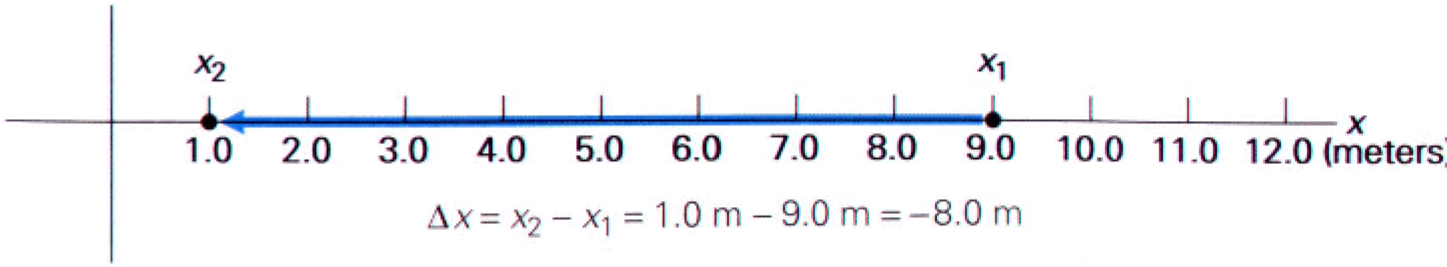
Suppose a person moves in a straight line from the lockers(at a position x = 1.0 m) toward the physics lab(at a position x = 9.0 m) , as shown below



The answer is positive so the person must have been traveling horizontally <u>to the right.</u>

Example

Suppose the person turns around!



The answer is negative so the person must have been traveling horizontally <u>to the left</u>

What is the *DISPLACEMENT* for the entire trip?

$$\Delta x = x_{final} - x_{initial} = 1.0 - 1.0 = 0m$$

What is the total *DISTANCE* for the entire trip?

$$8 + 8 = 16m$$

Average Velocity

Velocity is defined as: "The ***RATE*** at which ***DISPLACEMENT*** changes".

Rate = ANY quantity divided by ***TIME***.

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_{final} - x_{inital}}{t_{final} - t_{initial}}$$

Average ***SPEED*** is simply the "RATE at which DISTANCE changes".

$$\bar{s} = \frac{\Delta d}{\Delta t}$$

Example

A quarterback throws a pass to a defender on the other team who intercepts the football. Assume the defender had to run 50 m away from the quarterback to catch the ball, then 15 m towards the quarterback before he is tackled. The entire play took 8 seconds.

Let's look at the defender's average velocity:

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{35m - 0m}{8s - 0s} = 4.38\, m/s$$

Let's look at the defender's speed:

$$\bar{s} = \frac{d}{t} = \frac{65m}{8s} = 8.125\, m/s$$

"**m/s**" is the derived unit for both speed and velocity.

Instantaneous Velocity

Instantaneous velocity is a measure of an object's displacement per unit time at a particular point in time.

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

$$v = \frac{dx}{dt}$$

Example: A body's position is defined as:

$$x(t) = 7t^3\hat{i} - \frac{4}{t}\hat{j},\ v(t) = ?$$

$$v = \frac{dx}{dt} = \frac{d(7t^3\hat{i} - \frac{4}{t}\hat{j})}{dt}$$

→

$$v(t) = [21t^2\hat{i} + \frac{4}{t^2}\hat{j}]\,m/s$$

Average Acceleration

Acceleration is the RATE at which VELOCITY *changes*.

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_{final} - v_{initial}}{t_{final} - t_{initial}}$$

A truck accelerates from 10 m/s to 30 m/s in 2.0 seconds. What is the acceleration?

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{30m/s - 10m/s}{2s - 0s} = \frac{20m/s}{2s} = 10\ m/s/s\ or\ m/s^2$$

Suppose the same truck then slows down to 5 m/s in 4 seconds. What is the acceleration?

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{5m/s - 30m/s}{4s - 0s} = \frac{-25m/s}{2s} = -12.5\ m/s/s\ or\ m/s^2$$

"m/s/s" or "m/s^2" is the derived unit for acceleration.

What do the “signs”(+ or -) mean?

Quantity	Positive	Negative
Displacement	You are traveling north, east, right, or in the +x or +y direction.	You are traveling south, west, left, or in the –x or –y direction.
Velocity	The rate you are traveling north, east, right, or in the +x or +y direction.	The rate you are traveling south, west, left, or in the –x or –y direction.
Acceleration	Your velocity(speed) is increasing in a positive direction or your speed is decreasing in a negative direction.	Your velocity(speed) is decreasing in a positive direction or your speed is increasing in a negative direction.

Beware – the signs can confuse!

Suppose a ball is thrown straight upwards at 40 m/s. It takes 4 seconds to reach its maximum height, then another 4 seconds back down to the point where it was thrown. Assume it is caught with the same speed it was thrown. Calculate the acceleration upwards and downwards.

Upwards

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{0m/s - 40m/s}{4s - 0s} = \frac{-40m/s}{4s} = -10m/s$$

Downwards

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{-40m/s - 0m/s}{4s - 0s} = \frac{-40m/s}{4s} = -10m/s$$

This negative sign came from using the ***DELTA***

This negative sign came from the ***DIRECTION*** of the velocity.

It is no surprise you get a negative answer both ways as gravity acts DOWNWARDS no matter if the ball goes up or down. It is GRAVITY which changes the ball's velocity.

Instantaneous Acceleration

Instantaneous velocity is a measure of an object's velocity per unit time at a particular point in time.

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$$

$$a = \frac{dv}{dt}$$

If the velocity of an object is defined as:

$$v(t) = [21t^2\hat{i} + \frac{4}{t^2}\hat{j}]\,m/s,\ \ a(t) = ?$$

$$a = \frac{dv}{dt} = \frac{d(21t^2\hat{i} + \frac{4}{t^2}\hat{j})}{dt} = \ [42t\hat{i} - \frac{8}{t^3}\hat{j}]\,m/s/s$$