Internal Resistance and Resistivity in DC Circuits

AP Physics C
Internal Resistance

All components in a circuit offer some type of resistance regardless of how large or small it is. Batteries especially have what is called an **internal resistance**, \( r \).

Within the schematic it will be represented as a resistor symbol next to a battery symbol and between 2 points that represent the positive and negative terminals of the battery. Many times they are labeled with letters.

Since the battery is in effect a resistor, there is a voltage drop across it. Therefore there is only a certain amount of voltage that actually goes out to the circuit. That voltage is called the **terminal voltage**, \( V_T \).
To solve situations involving internal resistance we must use Kirchhoff's Voltage Law.

Going around the circuit counterclockwise.

We define the **maximum voltage that the battery can produce the EMF**.

Some of the voltage will DROP across the battery. The rest will drop ACROSS the external circuit. **This is called the terminal voltage.**

When KVL is re-arranged algebraically it looks like the slope of a line!
There are many graphical applications as the equation above looks like the slope intercept form of a line. The terminal voltage is plotted on the Y-axis, the current is plotted on the X-axis, the internal resistance is the SLOPE, the EMF is the Y-intercept.

\[ V_T = -rI + \varepsilon \]
\[ y = mx + b \]
Example

Suppose we have a car battery with an emf $= 13.8$ V, under a resistive load of $20 \, \Omega$, the voltage sags to $11.8$ V.

a) What is the battery's resistance?

$$V_T = IR_{Load}$$

$$11.8 = I(20)$$

$I = 0.58$ A

$$V_T = -rI + \varepsilon$$

$$11.8 = -r(?) + 13.8$$

$r = 3.45 \, \Omega$

b) What is the rate at which energy is dissipated in the battery?

$$P = VI$$

$$P = (2)(0.58) = 1.16 \, \text{W}$$

The car’s battery is in series with the load so the current is the SAME throughout the circuit.
Resistivity

All wires in a circuit also contribute to the overall resistance in a circuit. Even though the value is often small and negligible, it is often important to determine the value for the resistance of a wire if it is thick or long. This being said, the resistance is dependant on the geometry of the material

\[ R \alpha \ell \quad R \alpha \frac{1}{A} \]

\[ \rho = \text{Resistivity Constant} \]

\[ R = \frac{\rho \ell}{A} \]

The resistance of the wire is DIRECTLY proportional to the length and inversely proportional to the area. The constant of proportionality is then defined as the RESISTIVITY, which is based on material type.
Example

Calculate the resistance of a one meter length of 24 SWG Nichrome wire.

\[ R = \frac{\rho l}{A} = \frac{(1.10 \times 10^{-6})(1)}{\pi (2.795 \times 10^{-4})^2} = 4.48 \ \Omega \]

As you can see, using significant amounts of wire can greatly influence the voltage drops, current, and power produced in circuits.

\[ SWG = \text{Standard Wire Gauge} \]

\[ 24 \text{SWG} = 0.558 \text{mm} \text{ in diameter} \]

\[ \rho_{\text{Nichrome}} = 1.10 \times 10^{-6} \ \Omega \text{m} \]