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# ElectroMagnetic Induction

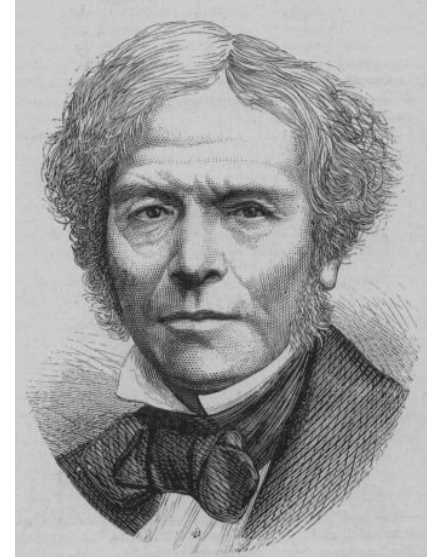
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AP Physics C

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# What is E/M Induction?

Electromagnetic Induction is the process of using magnetic fields to produce voltage, and in a complete circuit, a current.



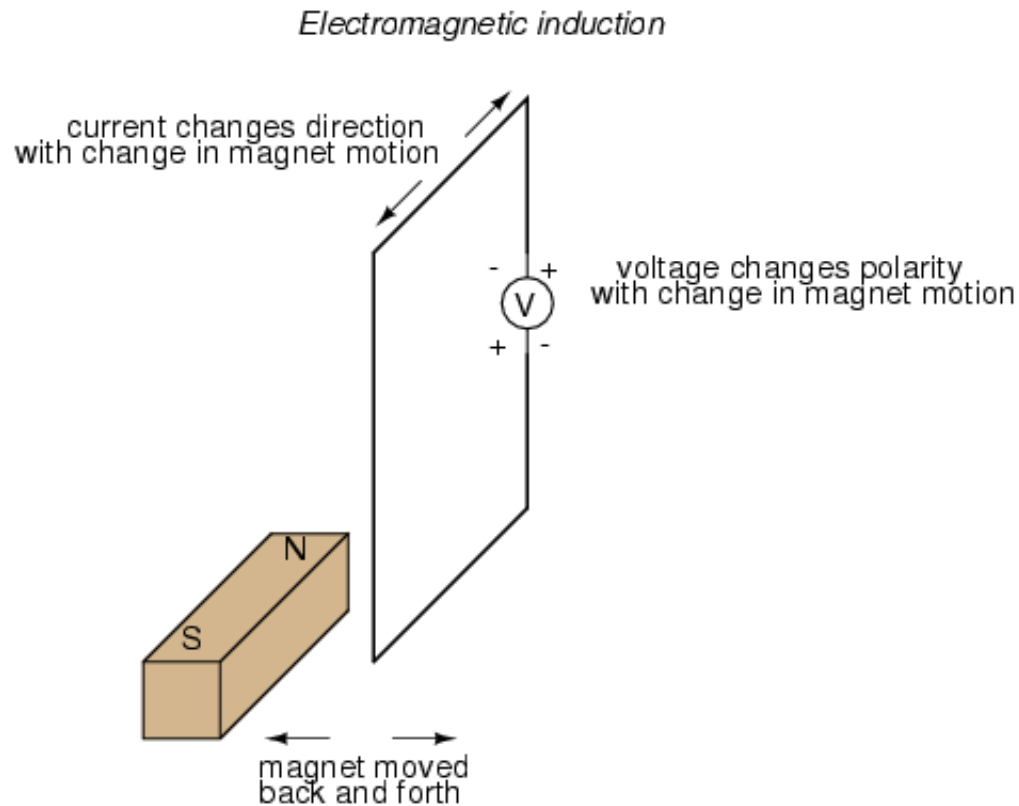
**Michael Faraday** first discovered it, using some of the works of Hans Christian Oersted. His work started at first using different combinations of wires and magnetic strengths and currents, but it wasn't until he tried moving the wires that he got any success.

It turns out that electromagnetic induction is created by just that - the moving of a conductive substance through a magnetic field.

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# Magnetic Induction

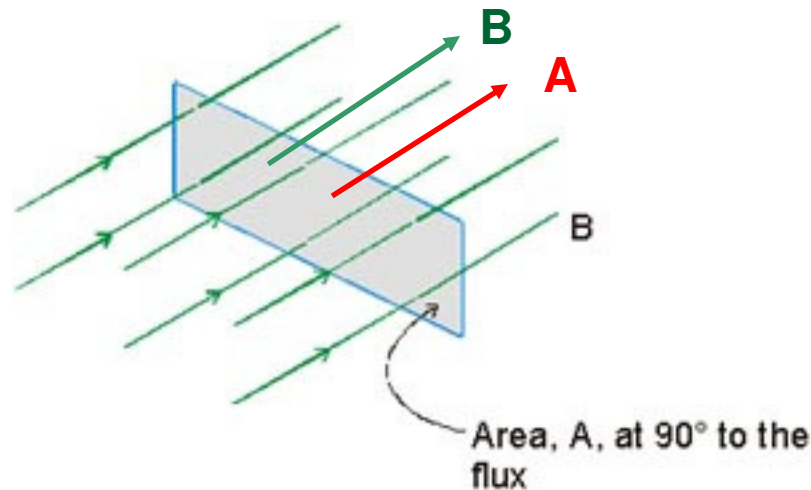
As the magnet **moves** back and forth a current is said to be INDUCED in the wire.



# Magnetic Flux

The first step to understanding the complex nature of electromagnetic induction is to understand the idea of magnetic flux.

Flux is a general term associated with a FIELD that is bound by a certain AREA. So **MAGNETIC FLUX** is any **AREA** that has a **MAGNETIC FIELD** passing through it.



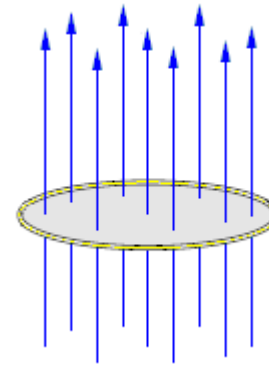
We generally define an AREA vector as one that is perpendicular to the surface of the material. Therefore, you can see in the figure that the AREA vector and the Magnetic Field vector are **PARALLEL**. This then produces a **DOT PRODUCT** between the 2 variables that then define flux.

# Magnetic Flux – The DOT product

$$\Phi_B = B \cdot A$$

$$\Phi_B = BA \cos \theta$$

Unit:  $\text{Tm}^2$  or Weber(Wb)



**How could we CHANGE the flux over a period of time?**

- We could move the magnet away or towards (or the wire)
- We could increase or decrease the area
- We could ROTATE the wire along an axis that is PERPENDICULAR to the field thus changing the angle between the area and magnetic field vectors.

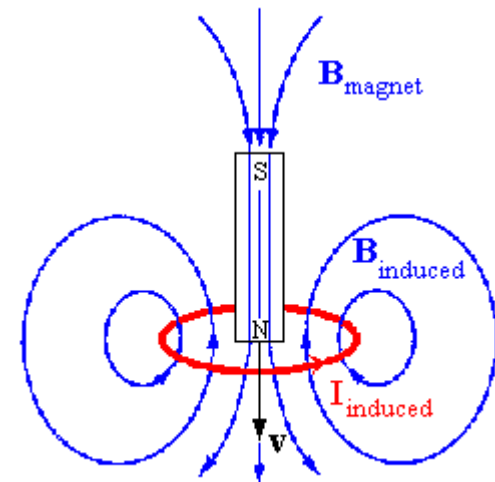
# Faraday's Law

Faraday learned that if you change any part of the flux over time you could induce a current in a conductor and thus create a source of EMF (voltage, potential difference). Since we are dealing with time here were a talking about the **RATE of CHANGE of FLUX**, which is called Faraday's Law.

$$\mathcal{E} = -N \frac{\Delta\Phi_B}{\Delta t} = -N \frac{\Delta(BA \cos \theta)}{\Delta t}$$

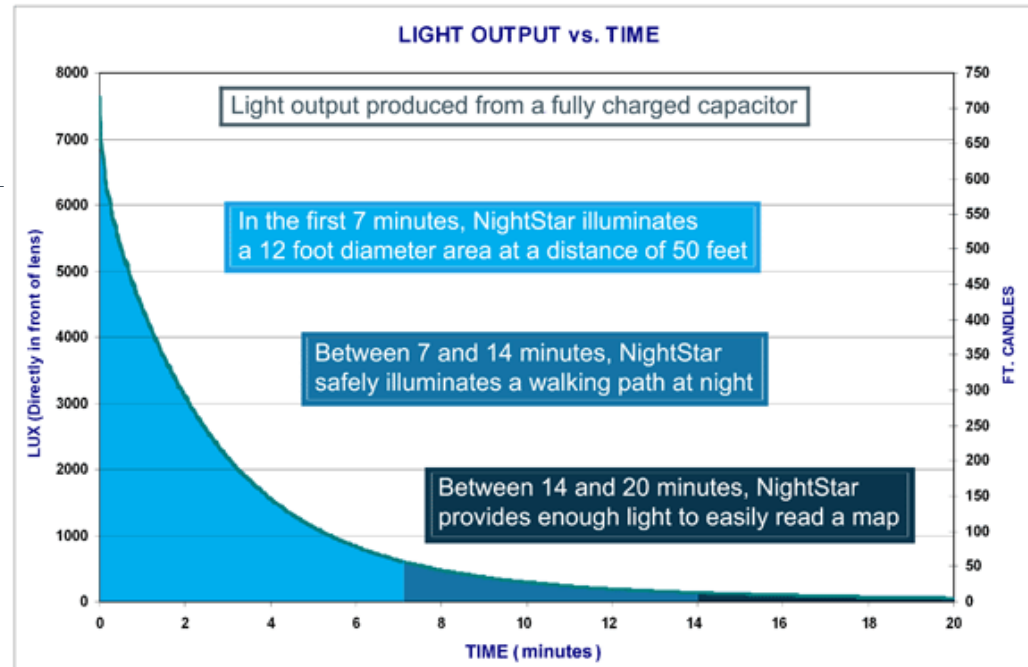
$N$  = # turns of wire

$$\mathcal{E} = -\frac{d\Phi_B}{dt} \quad \Phi_B = -\int \mathcal{E} dt$$

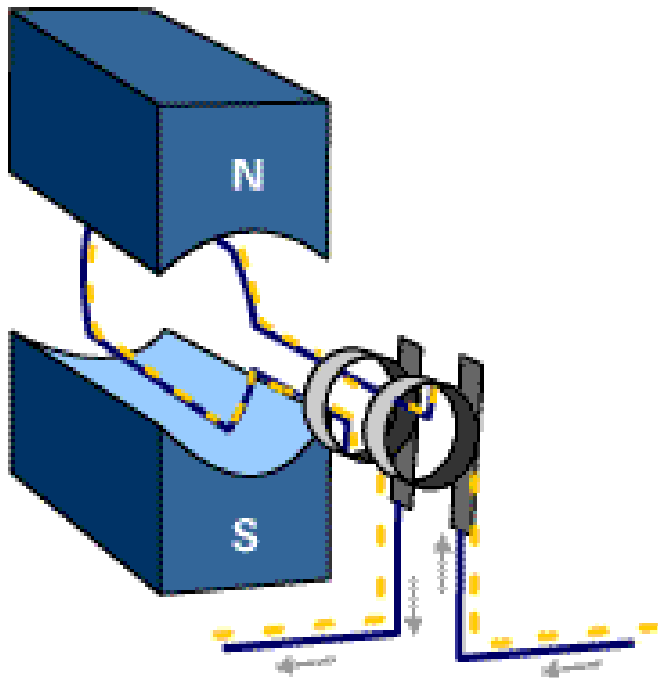


# Useful Applications

The Forever Flashlight uses the Faraday Principle of Electromagnetic Energy to eliminate the need for batteries. The Faraday Principle states that if an electric conductor, like copper wire, is moved through a magnetic field, electric current will be generated and flow into the conductor.



# Useful Applications

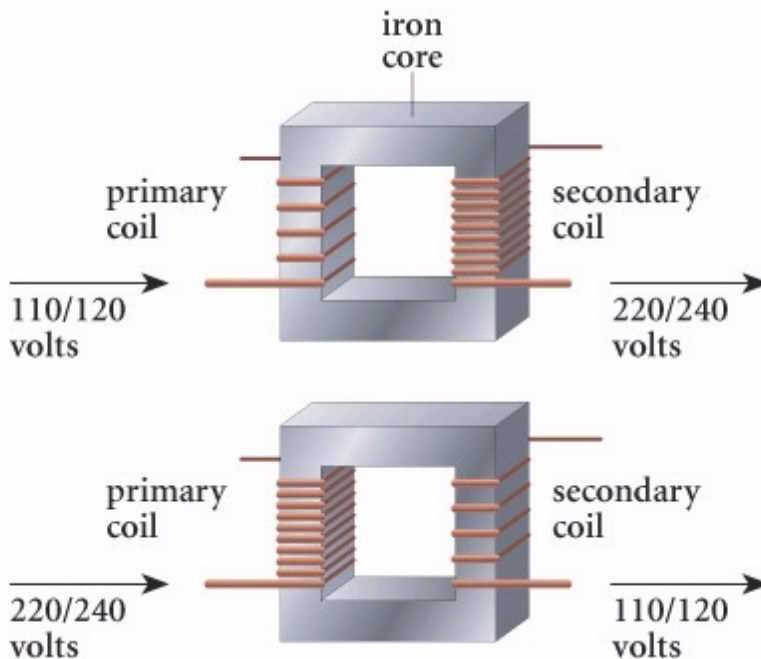


AC Generators use Faraday's law to produce rotation and thus convert electrical and magnetic energy into rotational kinetic energy. This idea can be used to run all kinds of motors. Since the current in the coil is **AC**, it is turning on and off thus creating a **CHANGING** magnetic field of its own. Its own magnetic field interferes with the shown magnetic field to produce rotation.



# Transformers

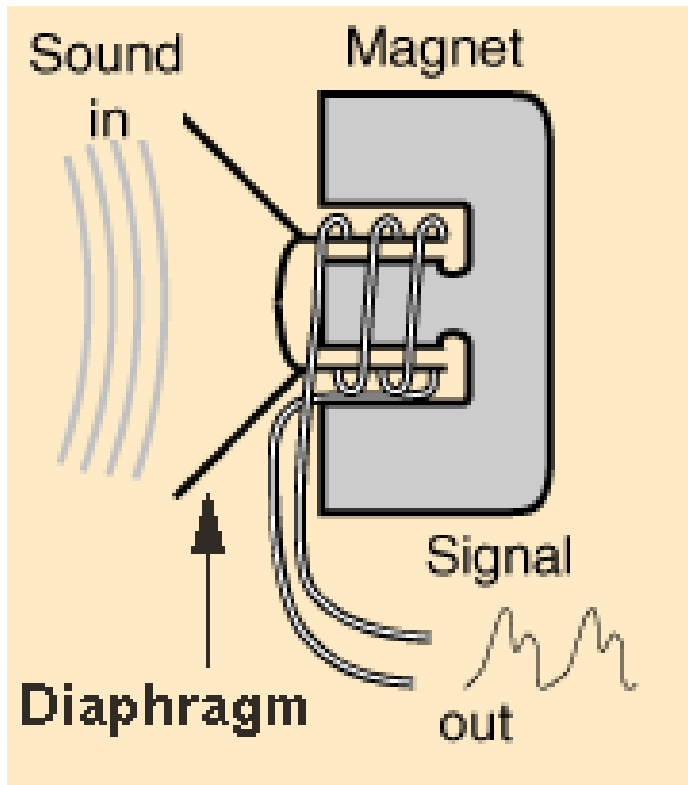
Probably one of the greatest inventions of all time is the transformer. AC Current from the primary coil moves quickly BACK and FORTH (thus the idea of changing!) across the secondary coil. The moving magnetic field caused by the changing field (flux) induces a current in the secondary coil.



If the secondary coil has MORE turns than the primary you can step up the voltage and runs devices that would normally need MORE voltage than what you have coming in. We call this a STEP UP transformer.

We can use this idea in reverse as well to create a STEP DOWN transformer.

# Microphones



A microphone works when sound waves enter the filter of a microphone. Inside the filter, a diaphragm is vibrated by the sound waves which in turn moves a coil of wire wrapped around a magnet. The movement of the wire in the magnetic field induces a current in the wire. Thus sound waves can be turned into electronic signals and then amplified through a speaker.

# Example

A coil with 200 turns of wire is wrapped on an 18.0 cm square frame. Each turn has the same area, equal to that of the frame, and the total resistance of the coil is  $2.0\Omega$ . A uniform magnetic field is applied perpendicularly to the plane of the coil. If the field changes uniformly from 0 to 0.500 T in 0.80 s, find the magnitude of the induced emf in the coil while the field has changed as well as the magnitude of the induced current.

$$|\mathcal{E}| = N \frac{\Delta\Phi_B}{\Delta t} = N \frac{\Delta BA \cos \theta}{\Delta t}$$

$$|\mathcal{E}| = 200 \frac{(0.500 - 0)(0.18 \times 0.18) \cos 90}{0.80}$$

$$|\mathcal{E}| = \mathbf{4.05 \text{ V}}$$

$$\mathcal{E} = IR = I(2)$$

$$I = \mathbf{2.03 \text{ A}}$$

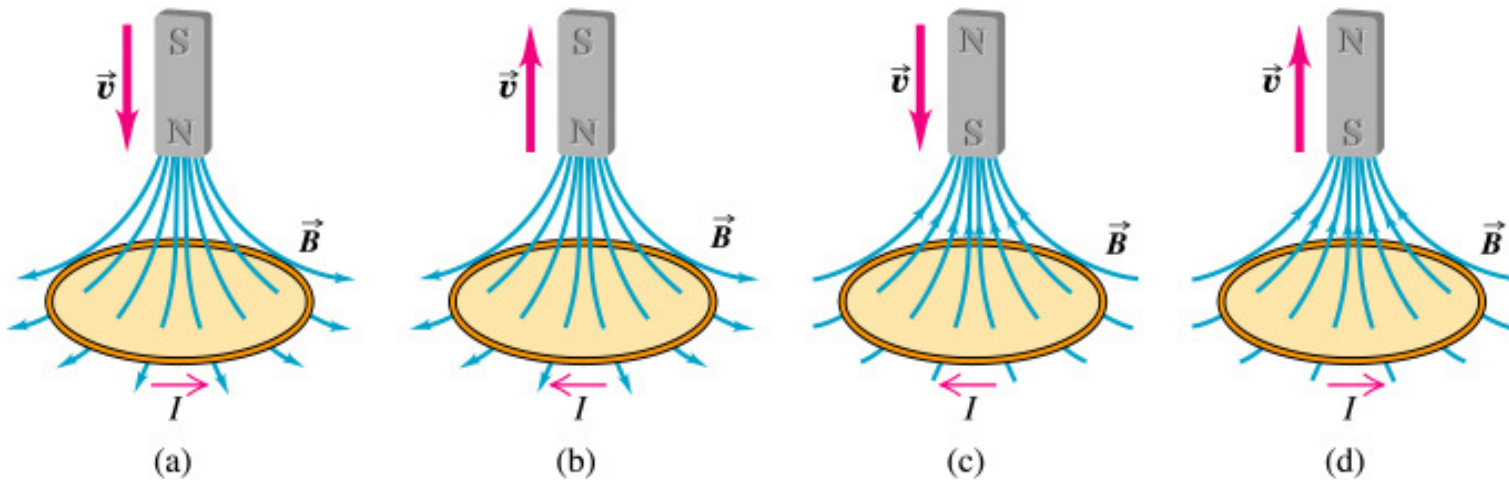
**Why did you find the ABSOLUTE VALUE of the EMF?**

**What happened to the “ - “ that was there originally?**

# Lenz's Law

Lenz's law gives the **direction** of the induced emf and current resulting from electromagnetic induction. The law provides a physical interpretation of the **choice of sign** in Faraday's law of induction, indicating that the induced emf and the change in flux have **opposite signs**.

Lenz's Law  $\mathcal{E} = -N \frac{\Delta\Phi_B}{\Delta t}$

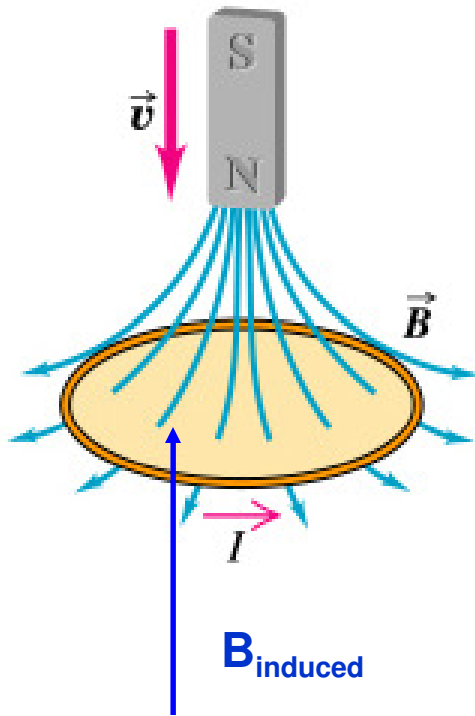


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In the figure above, we see that the direction of the current changes. Lenz's Law helps us determine the **DIRECTION** of that current.

# Lenz's Law & Faraday's Law

$$\mathcal{E} = -N \frac{\Delta\Phi_B}{\Delta t}$$



Let's consider a magnet with its north pole moving TOWARDS a conducting loop.

DOES THE FLUX CHANGE? **Yes!**

DOES THE FLUX INCREASE OR DECREASE?

**Increase**

WHAT SIGN DOES THE "Δ" GIVE YOU IN FARADAY'S LAW?

**Positive**

DOES LENZ'S LAW CANCEL OUT? **NO**

**What does this mean?**

This means that the INDUCED MAGNETIC FIELD around the WIRE caused by the moving magnet OPPOSES the original magnetic field. Since the original B field is downward, the induced field is upward! We then use the curling right hand rule to determine the direction of the current.

# Lenz's Law

The INDUCED current creates an INDUCED magnetic field of its own inside the conductor that opposes the original magnetic field.

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A magnet is dropped down a conducting tube.

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The magnet INDUCES a current above and below the magnet as it moves.

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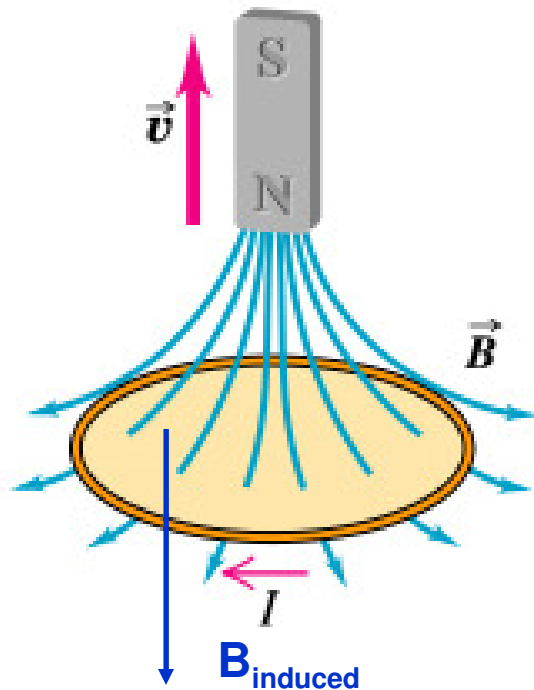


Since the induced field opposes the direction of the original it attracts the magnet upward slowing the motion caused by gravity downward.

If the motion of the magnet were NOT slowed this would violate conservation of energy!

# Lenz's Law

$$\mathcal{E} = -N \frac{\Delta\Phi_B}{\Delta t}$$



Let's consider a magnet with its north pole moving AWAY from a conducting loop.

DOES THE FLUX CHANGE? **Yes!**

DOES THE FLUX INCREASE OR DECREASE?  
**Decreases**

WHAT SIGN DOES THE "Δ" GIVE YOU IN FARADAY'S LAW?  
**negative**

DOES LENZ'S LAW CANCEL OUT? **yes**

**What does this mean?**

In this case, the induced field DOES NOT oppose the original and points in the same direction. Once again use your curled right hand rule to determine the DIRECTION of the current.

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## In summary

**Faraday's Law** is basically used to find the MAGNITUDE of the induced EMF. The magnitude of the current can then be found using Ohm's Law provided we know the conductor's resistance.

**Lenz's Law** is part of Faraday's Law and can help you determine the direction of the current provided you know HOW the flux is changing

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# Example

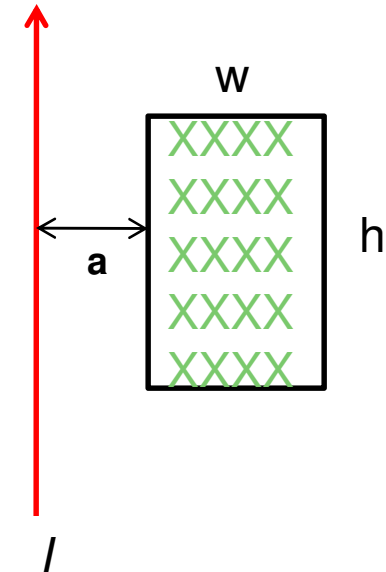
A long, straight wire carrying a current,  $I$  is placed near a loop of dimensions  $w$  and  $h$  as shown. Calculate the magnetic flux for this loop.

What is the direction of the magnetic field inside the loop due to the current carrying wire?

Into the page

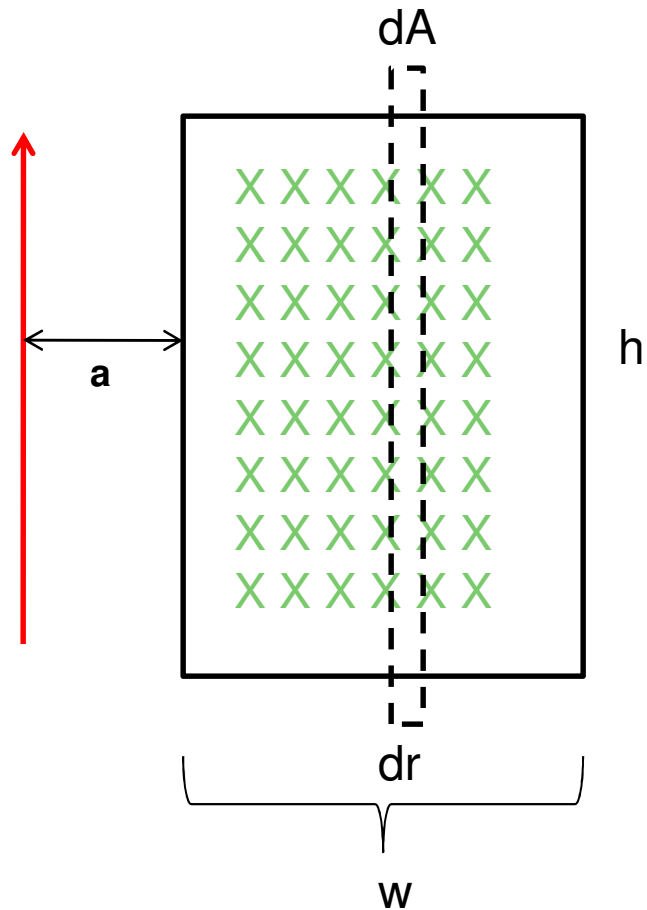
$$\Phi = BA \cos \theta \quad B_{\text{wire}} = \frac{\mu_o I}{2\pi a} \quad A = wh$$

$$\Phi = \frac{\mu_o I wh}{2\pi a}$$



BUT...here is the problem. The spacial uniformity IS NOT the same as you move away from the wire. The magnetic field CHANGES, or in this case decreases, as you move away from the wire the FLUX changes. So the formula above does NOT illustrate the correct function for the flux

# Example



You begin by taking a slice of the area. In other words, begin with a differential amount of AREA,  $dA$ , that is a differential amount of distance wide, which we will call,  $dr$ .

Then we must think about our limits. We need to SUM all of the area starting at "a" and going to "w+a".

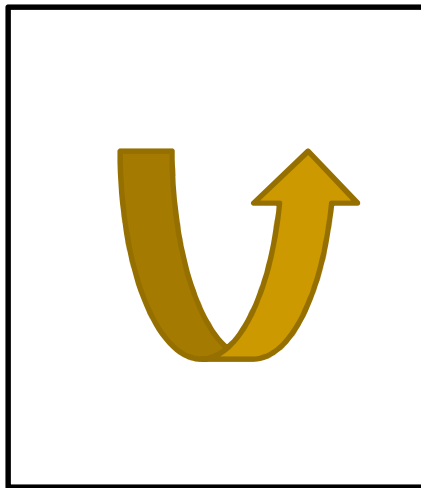
$$\Phi = BA \cos \theta \quad B_{\text{wire}} = \frac{\mu_o I}{2\pi r} \quad \Phi = \frac{\mu_o I}{2\pi r} dA \quad dA = h dr$$

$$\Phi = \int_a^{w+a} \frac{\mu_o I h}{2\pi r} dr \rightarrow \frac{\mu_o I h}{2\pi} \int_a^{w+a} \frac{1}{r} dr$$

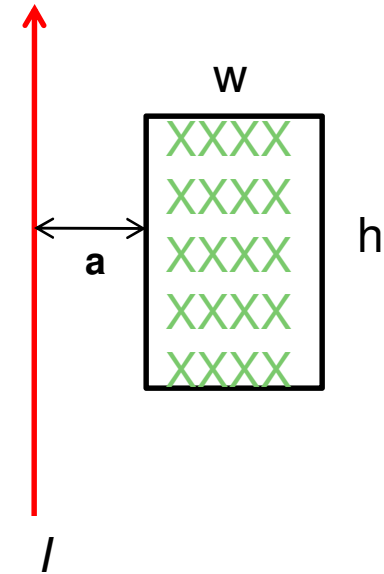
$$\Phi = \frac{\mu_o I h}{2\pi} \ln\left(\frac{w+a}{a}\right)$$

# Example

If the loop is moving TOWARDS the wire, what is the direction of the “induced” current around the loop?

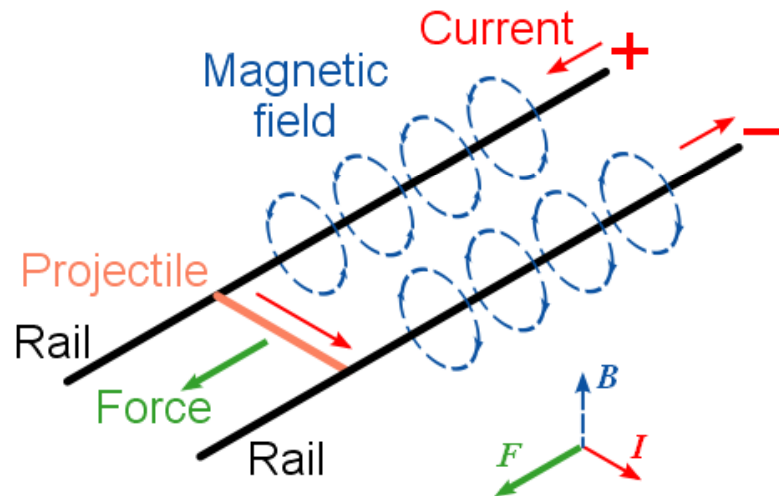


Since the original field is INTO THE PAGE and the FLUX INCREASES, the negative sign (Lenz’s Law) in Faraday’s Law remains and therefore the induced field is in the opposite direction to oppose the change, which is OUT OF THE PAGE. This produces a current which is **counter-clockwise** around the loop



# Motional EMF – The Rail Gun

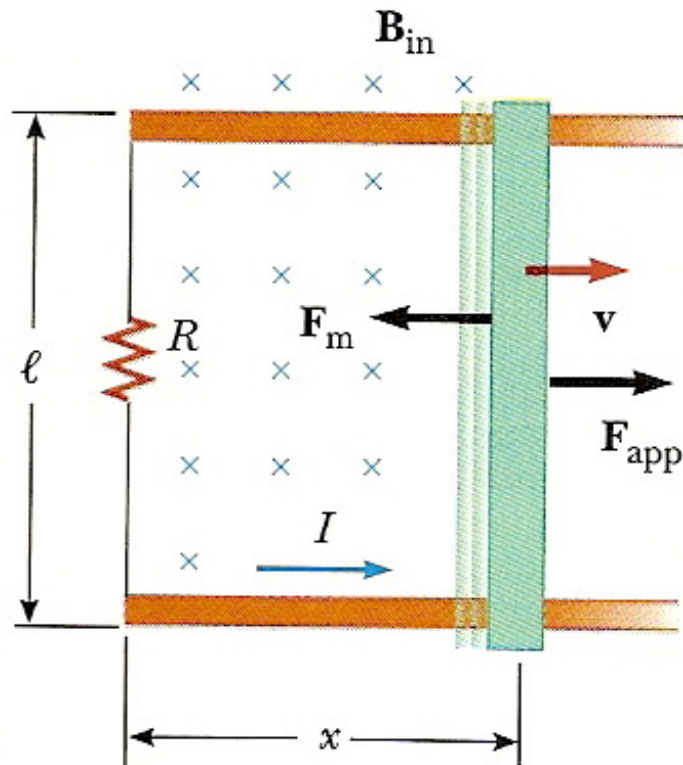
A railgun consists of two parallel metal rails (hence the name) connected to an electrical power supply. When a conductive projectile is inserted between the rails (from the end connected to the power supply), it completes the circuit. Electrons flow from the negative terminal of the power supply up the negative rail, across the projectile, and down the positive rail, back to the power supply.



In accordance with the right-hand rule, the magnetic field circulates around each conductor. Since the current is in opposite direction along each rail, the net magnetic field between the rails ( $\mathbf{B}$ ) is directed vertically. In combination with the current ( $\mathbf{I}$ ) across the projectile, this produces a magnetic force which accelerates the projectile along the rails. There are also forces acting on the rails attempting to push them apart, but since the rails are firmly mounted, they cannot move. The projectile slides up the rails away from the end with the power supply.

# Motional Emf

There are many situations where motional EMF can occur that are different from the rail gun. Suppose a bar of length,  $L$ , is pulled to right at a speed,  $v$ , in a magnetic field,  $B$ , directed into the page. The conducting rod itself completes a circuit across a set of parallel conducting rails with a resistor mounted between them.



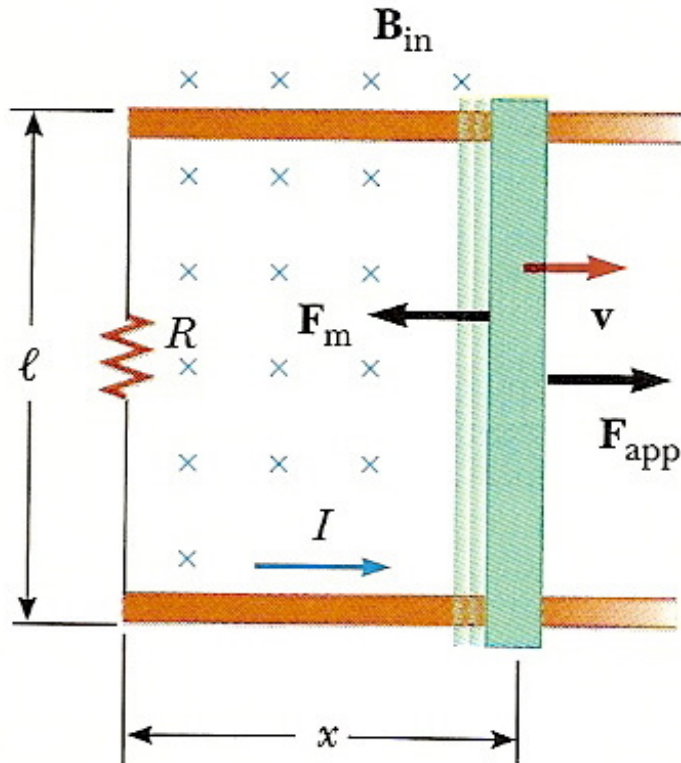
$$\mathcal{E} = -N \frac{\Delta\Phi_B}{\Delta t}$$

$$\mathcal{E} = \frac{BA}{t} \rightarrow \frac{Blx}{t}; \quad \boxed{\mathcal{E} = Blv}$$

$$\mathcal{E} = IR$$

$$I = \frac{Blv}{R}$$

# Motional EMF



In the figure, we are applying a force this time to the rod. Due to Lenz's Law the magnetic force opposes the applied force. Since we know that the magnetic force acts to the left and the magnetic field acts into the page, we can use the RHR to determine the direction of the current around the loop and the resistor.

# Example

An airplane with a wing span of 30.0 m flies parallel to the Earth's surface at a location where the downward component of the Earth's magnetic field is  $0.60 \times 10^{-4}$  T. Find the difference in potential between the wing tips if the speed of the plane is 250 m/s.

$$\mathcal{E} = Blv$$

$$\mathcal{E} = 0.60 \times 10^{-4} (30)(250)$$

$$\mathcal{E} = \mathbf{0.45 \text{ V}}$$



In 1996, NASA conducted an experiment with a 20,000-meter conducting tether. When the tether was fully deployed during this test, the orbiting tether generated a potential of 3,500 volts. This conducting single-line tether was severed after five hours of deployment. It is believed that the failure was caused by an electric arc generated by the conductive tether's movement through the Earth's magnetic field.

# Inductance

After investigating with Faraday's Law, we see that the magnetic flux is DIRECTLY related to the current. The proportionality constant in this case is called INDUCTANCE,  $L$ , which is a type of magnetic resistance. The unit of inductance is the HENRY, H.

$$\Phi_B \propto I$$

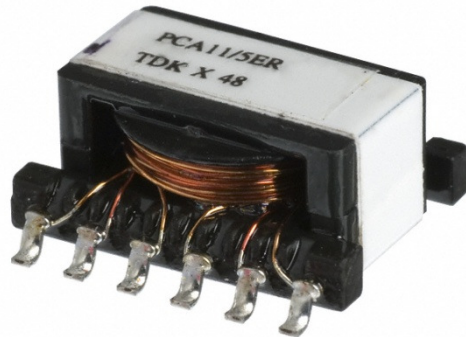
$L$  = constant of proportionality

$$\Phi_B = LI$$

If you divide both side by time we get:

$$\frac{\Phi_B}{t} = L \frac{I}{t} \quad \mathcal{E} = -\frac{d\Phi}{dt} = -L \frac{dI}{dt}$$

$$\mathcal{E} = -L \frac{dI}{dt}$$





# Inductance

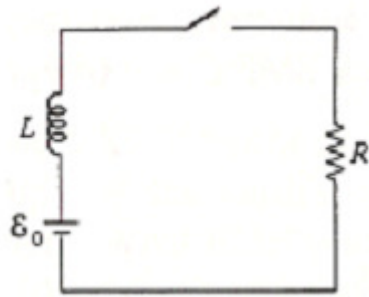
$$L = \frac{\Phi_B}{I} = \text{self-inductance}$$

$$\varepsilon = -L \frac{dI}{dt} = \text{Back EMF}$$

So what happens when we hook up a giant coil of wire to a circuit? We throw the switch and the current flows. The circuit will try to resist the change in flux as a result of the current. This is called BACK EMF! Usually the back EMF is very small so we don't need to worry about it. BUT, if there is a coil of wire the effect is VERY STRONG! If a current creates a magnetic flux in any circuit element we define this as SELF INDUCTANCE, L. The unit for inductance is henries.

What this tells us is HOW LARGE an INDUCED EMF we can expect across the coils of an inductor per change in current per unit time.

# Inductors in a circuit



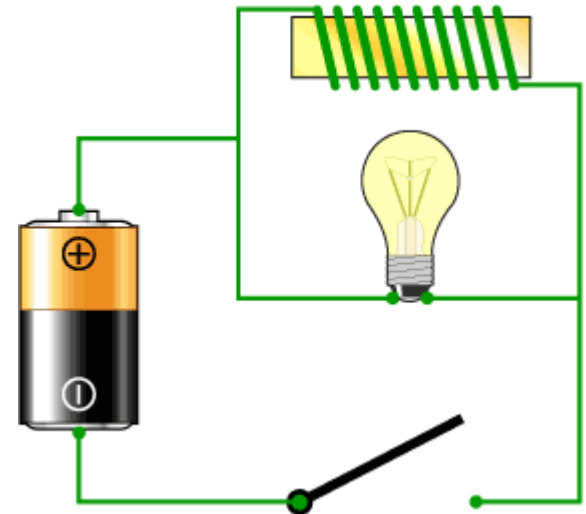
$$\epsilon_0 - L \frac{dI}{dt} - IR = 0$$

Multiply by I

$$\epsilon_0 I - LI \frac{dI}{dt} - I^2 R = 0, P = \frac{dU}{dt}$$

$$U_B = \int_0^t LI \frac{dI}{dt} dt = \int_0^I LI dI = L \int_0^I I dI$$

$$U_B = \frac{1}{2} LI^2$$



Using Kirchhoff's voltage law we have:

What we have now is **MAGNETIC ENERGY** stored in an **INDUCTOR**! This is very similar to a capacitor storing charge and producing electrical potential energy.