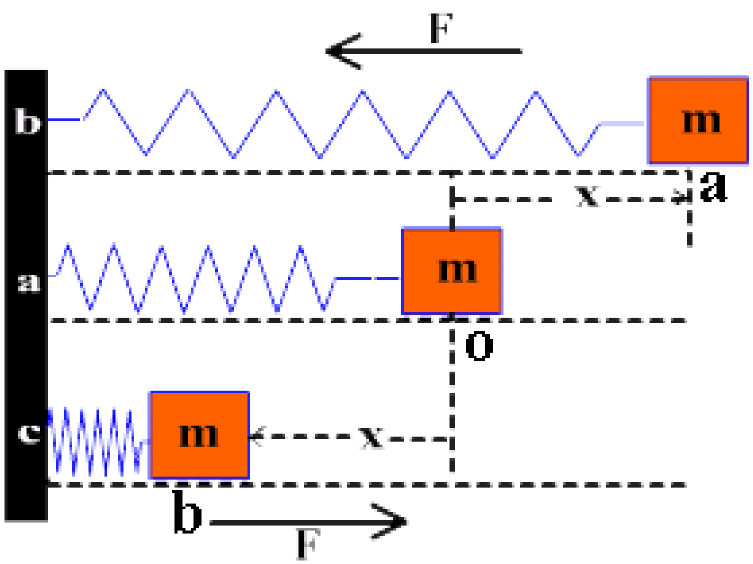
Elastic Potential Energy & Springs

AP Physics C

Simple Harmonic Motion

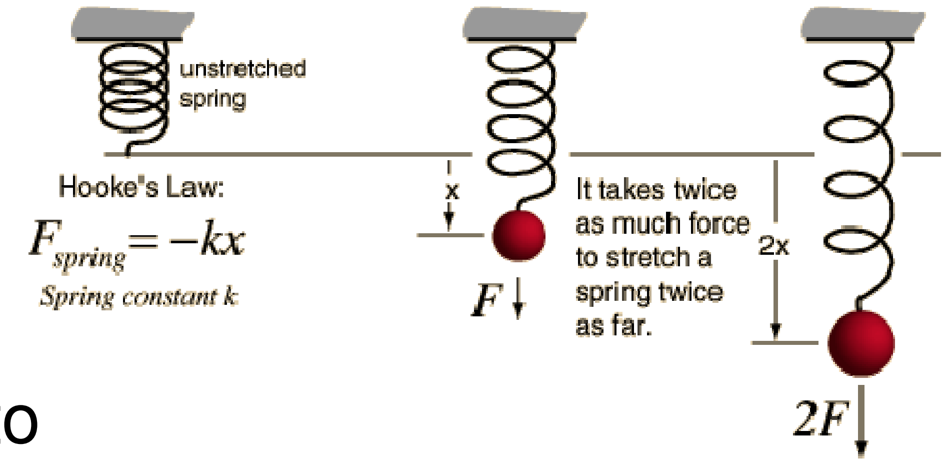
Back and forth motion that is caused by a force that is **directly proportional** to the displacement. The displacement centers around an equilibrium position.



$$F_s \alpha \; x \, (or \; \vec{r})$$

Springs – Hooke's Law

One of the simplest type of simple harmonic motion is called **Hooke's Law.** This is primarily in reference to SPRINGS.



$$F_s \,\alpha\, x$$

$$k = \text{Constant of Proportionality}$$

$$k = \text{Spring Constant(Unit : N/m)}$$

$$F_s = kx \quad or \quad -kx$$

The negative sign only tells us that "F" is what is called a RESTORING FORCE, in that it works in the OPPOSITE direction of the displacement.

Hooke's Law

Common formulas which are set equal to Hooke's law are N.S.L. and weight

$$F_s = kx$$

$$F_s = F_g \rightarrow kx = mg$$

$$F_s = F_{net} \rightarrow kx = ma$$

Example

A load of 50 N attached to a spring hanging vertically stretches the spring 5.0 cm. The spring is now placed horizontally on a table and stretched 11.0 cm. What force is required to stretch the spring this amount?

$$F_s = kx$$

$$50 = k(0.05)$$

$$k = \textbf{1000 N/m}$$

$$F_s = kx$$

$$F_s = (1000)(0.11)$$

$$F_s = \textbf{110 N}$$

Hooke's Law from a Graphical Point of View

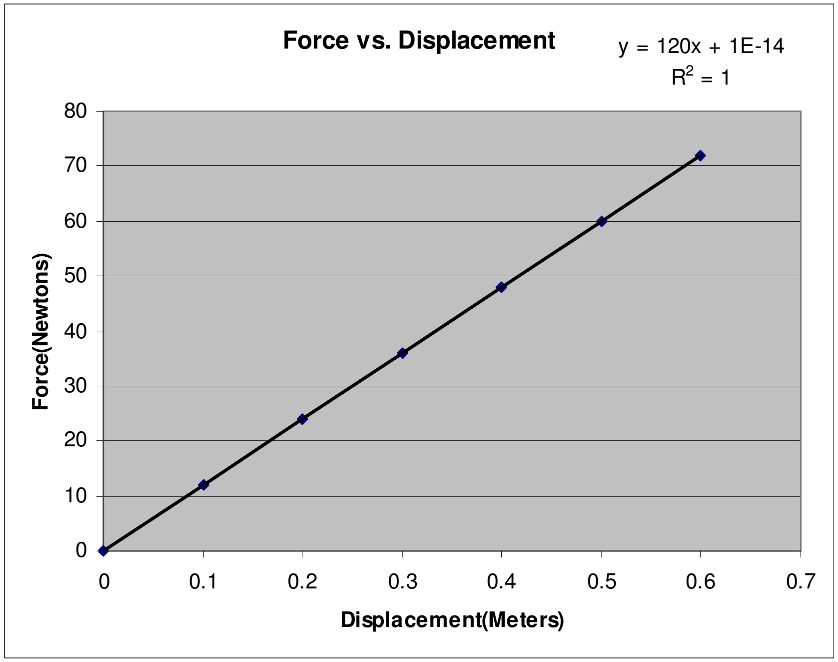
Suppose we had the following data:

x(m)	Force(N)
0	0
0.1	12
0.2	24
0.3	36
0.4	48
0.5	60
0.6	72

$$F_s = kx$$

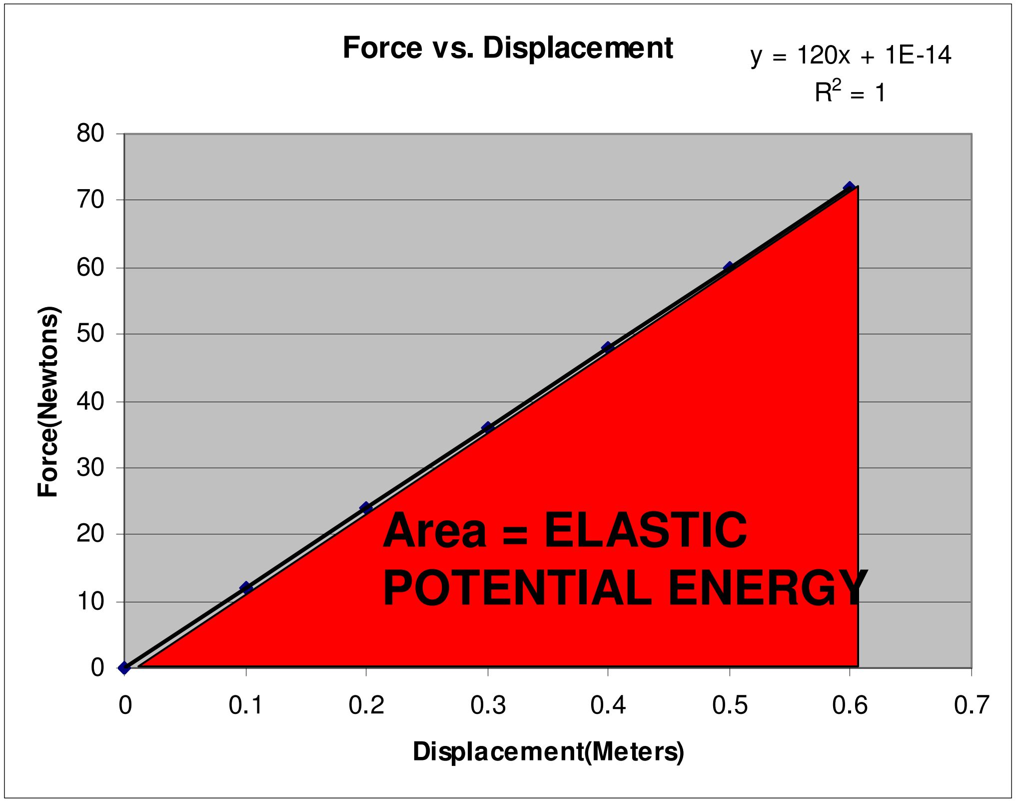
$$k = \frac{F_s}{x}$$

$$k = \text{Slope of a F vs. x graph}$$



k =120 N/m

We have seen F vs. x Before!!!!



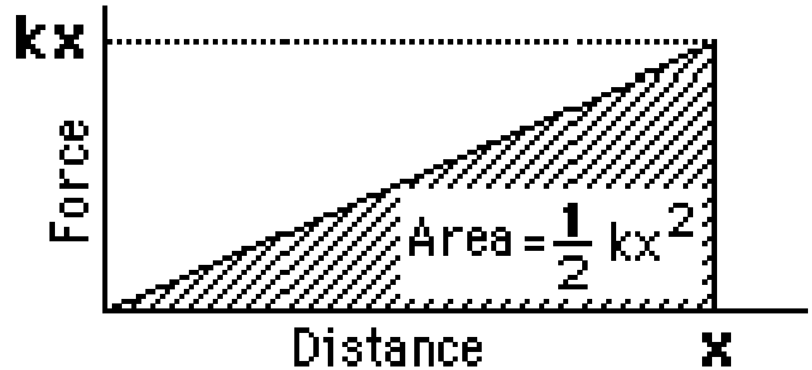
Work or ENERGY = $F\Delta x$

Since WORK or ENERGY is the AREA, we must get some type of energy when we compress or elongate the spring. This energy is the AREA under the line!

Since we STORE energy when the spring is compressed and elongated it classifies itself as a "type" of POTENTIAL ENERGY, U_s. In this case, it is called ELASTIC POTENTIAL ENERGY.

Elastic Potential Energy

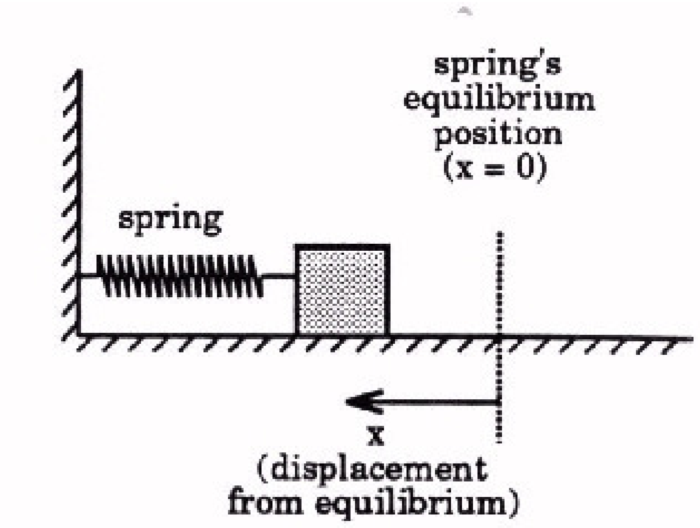
The graph of F vs.x for a spring that is IDEAL in nature will always produce a line with a positive linear slope. Thus the area under the line will always be represented as a triangle.



$$U_s = \frac{1}{2}kx^2$$

NOTE: Keep in mind that this can be applied to **WORK** or can be conserved with any other type of energy.

Elastic potential energy



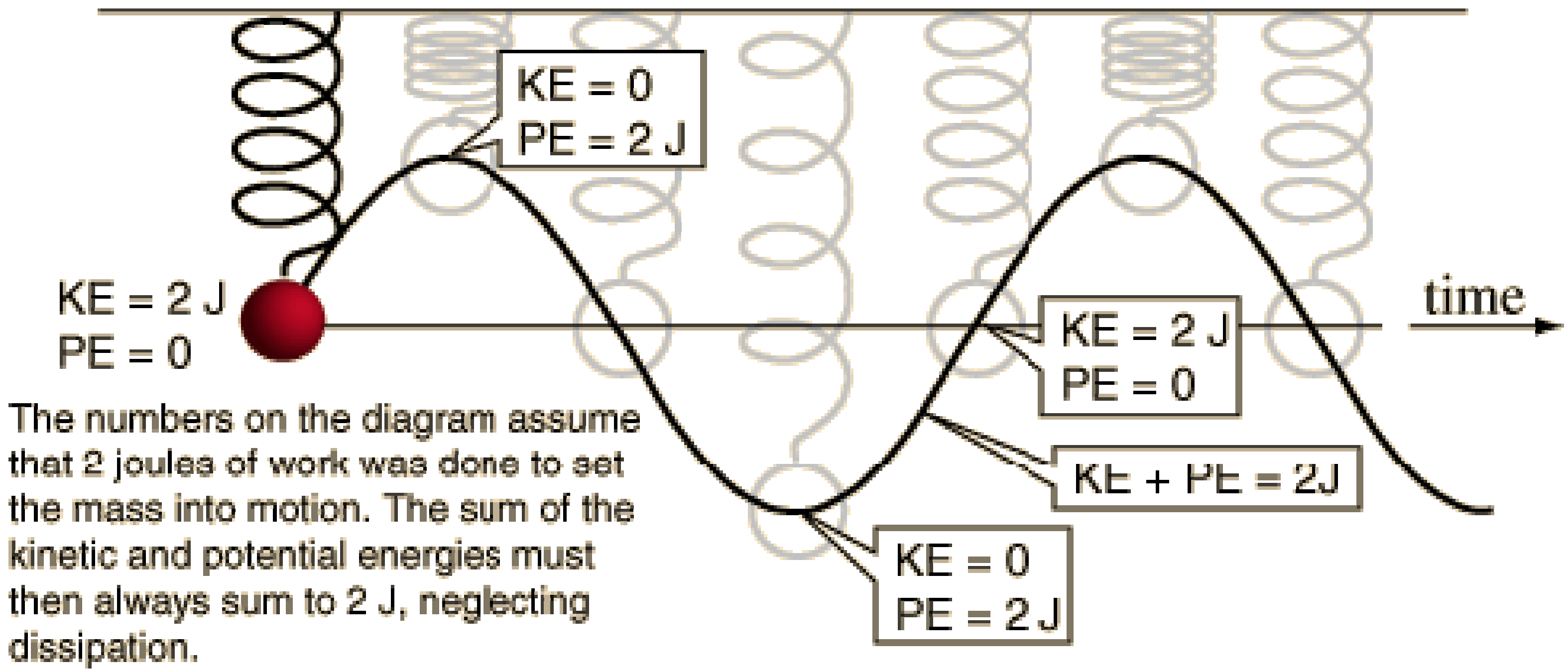
$$W = \int F(x)dx \rightarrow \int (kx)dx$$

$$W = \int_{x=0}^{x} (kx)dx \rightarrow k\int_{x=0}^{x} xdx$$

$$W = k \,|\, \frac{x^2}{2} \,|_{x=0}^{x} \rightarrow W = U_{spring} = \tfrac{1}{2} kx^2$$

Elastic "potential" energy is a fitting term as springs **STORE** energy when there are elongated or compressed.

Conservation of Energy in Springs



Example

A slingshot consists of a light leather cup, containing a stone, that is pulled back against 2 rubber bands. It takes a force of 30 N to stretch the bands 1.0 cm (a) What is the potential energy stored in the bands when a 50.0 g stone is placed in the cup and pulled back 0.20 m from the equilibrium position? (b) With what speed does it leave the slingshot?

$a)\ F_s = kx \quad 30 = k(0.01) \quad k =$ **3000 N/m**

$b)\ U_s = \frac{1}{2}kx^2 = 0.5(k)(.20) =$ **300 J**

$c)\ E_B = E_A \quad U_s = K$

$U_s = \frac{1}{2}mv^2 = \frac{1}{2}(0.050)v^2$

$v =$ **109.54 m/s**