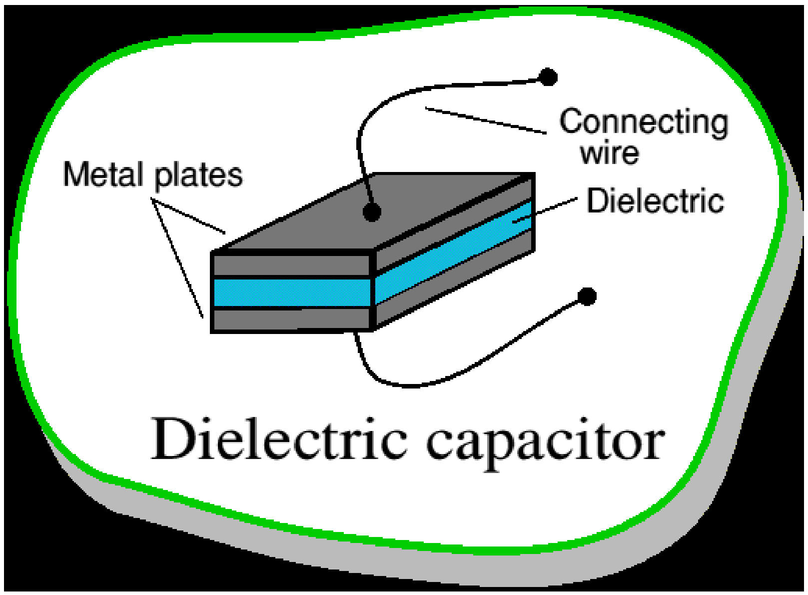
Capacitance and Dielectrics

AP Physics C

Applications of Electric Potential

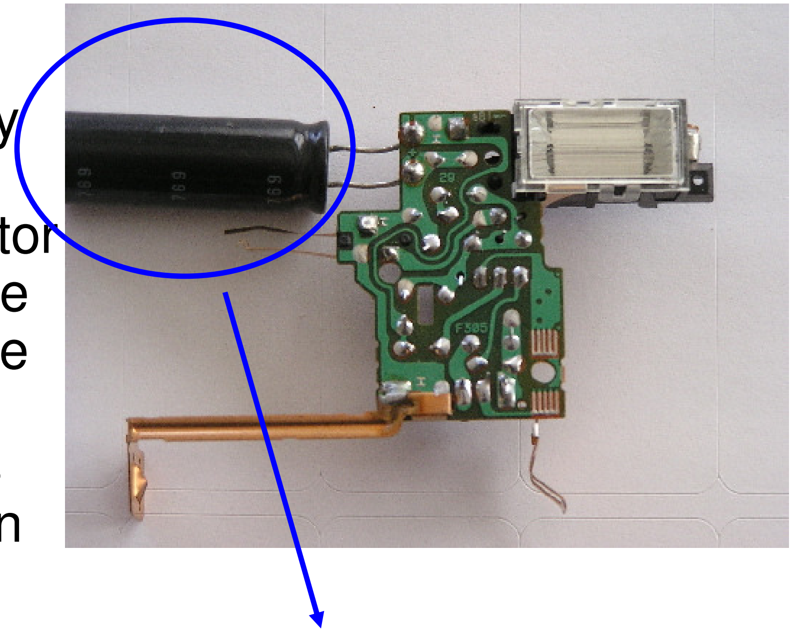
Is there any way we can use a set of plates with an electric field? YES! We can make what is called a Parallel Plate Capacitor and Store Charges between the plates!



Storing Charges- Capacitors
A capacitor consists of 2 conductors of any shape placed near one another without touching. It is common; to fill up the region between these 2 conductors with an insulating material called a dielectric. We charge these plates with opposing charges to set up an electric field.

Capacitors in Kodak Cameras

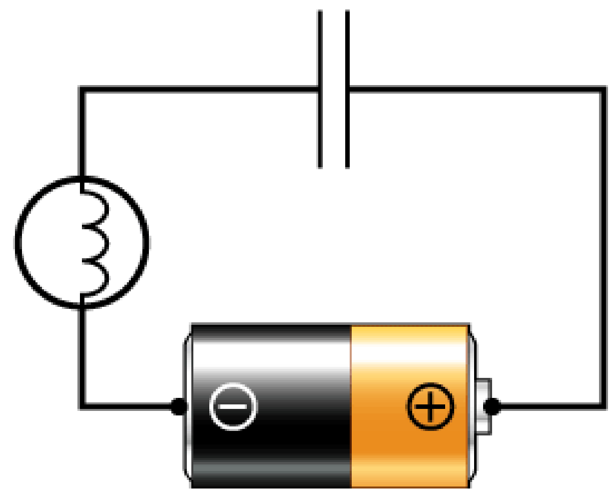
Capacitors can be easily purchased at a local Radio Shack and are commonly found in disposable Kodak Cameras. When a voltage is applied to an empty capacitor, current flows through the capacitor and each side of the capacitor becomes charged. The two sides have equal and opposite charges. When the capacitor is fully charged, the current stops flowing. The collected charge is then ready to be discharged and when you press the flash it discharges very quickly released it in the form of light.



Cylindrical Capacitor

Capacitance

In the picture below, the capacitor is symbolized by a set of parallel lines. Once it's charged, the capacitor has the same voltage as the battery (1.5 volts on the battery means 1.5 volts on the capacitor) The difference between a capacitor and a battery is that a capacitor can dump its entire charge in a tiny fraction of a second, where a battery would take minutes to completely discharge itself. That's why the electronic flash on a camera uses a capacitor -- the battery charges up the flash's capacitor over several seconds, and then the capacitor dumps the full charge into the flash tube almost instantly



Electric Potential for Conducting Sheets

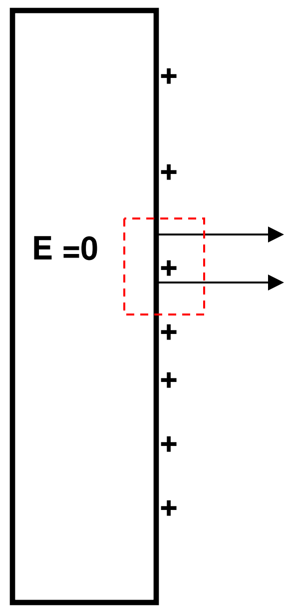
$$\oint E \bullet dA = \frac{q_{enc}}{\varepsilon_o}$$

$$EA = \frac{Q}{\varepsilon_o}$$

$$\sigma = \frac{Q}{A}, \quad EA = \frac{\sigma A}{\varepsilon_o}$$

$$E = \frac{\sigma}{\varepsilon_o}$$

Using Gauss' Law we derived and equation to define the electric field as we move radially away from the charged sheet or plate. Electric Potential?



$$\Delta V = -\int E\,dr$$

$$V(b) - V(a) = -\int_a^b \left(\frac{\sigma}{\varepsilon_o}\right) dr$$

$$V(b) - V(a) = \int_b^a \left(\frac{\sigma}{\varepsilon_o}\right) dr$$

$$V(b) - V(a) = \frac{\sigma}{\varepsilon_o}(a-b), \quad a-b=d$$

$$\Delta V = \frac{\sigma}{\varepsilon_o} d = Ed = \frac{Qd}{\varepsilon_o A}$$

This expression will be particularly useful later

Measuring Capacitance

Let's go back to thinking about plates!

$$\Delta V = Ed,$$

$$\Delta V \, \alpha \, E, if \;\; d = \text{constant}$$

$$E \, \alpha \, Q \quad\quad Therefore$$

$$Q \, \alpha \, \Delta V$$

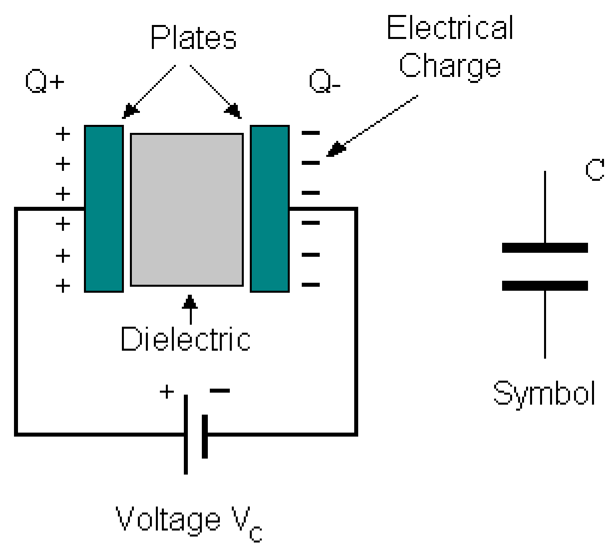
$$C = \text{contant of proportionality}$$

$$\text{C} = \text{Capacitance}$$

$$Q = CV$$

$$C = \frac{Q}{V}$$

The unit for capacitance is the **FARAD, F.**



Capacitance

$$\Delta V = \frac{\sigma}{\varepsilon_o} d = Ed = \frac{Qd}{\varepsilon_o A}$$

This was derived from integrating the Gauss' Law expression for a conducting plate.

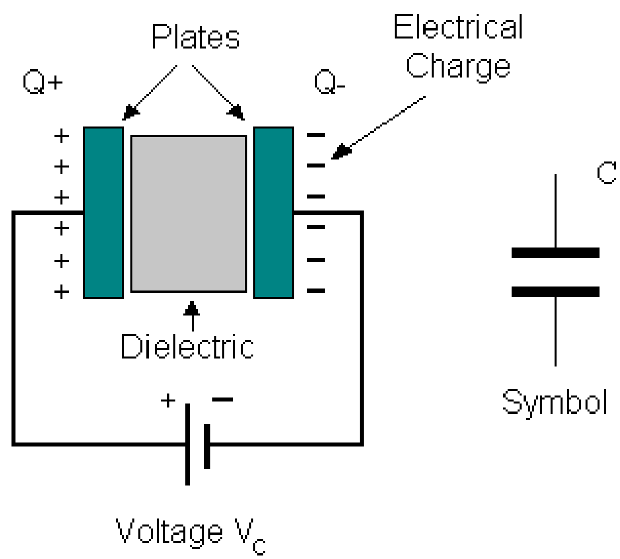
$$\Delta V = \left(\frac{d}{\varepsilon_o A}\right) Q \rightarrow \quad Q = \left(\frac{\varepsilon_o A}{d}\right) \Delta V$$

These variables represent a constant of proportionality between voltage and charge.

$$Q = C\Delta V$$

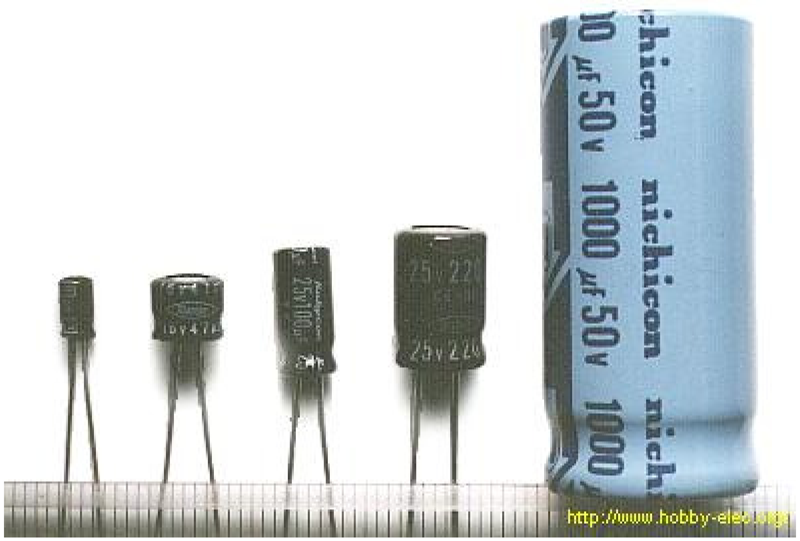
$$C = \frac{\varepsilon_o A}{d}$$

What this is saying is that YOU CAN change the capacitance even though it represents a constant. That CHANGE, however, can only happen by physically changing the GEOMETRY of the capacitor itself.



Capacitor Geometry

The capacitance of a capacitor depends on HOW you make it.



$$C \alpha A \quad C \alpha \frac{1}{\mathrm{d}}$$

$$A = \text{area of plate}$$

$$\mathrm{d} = \text{distance beteween plates}$$

$$C \alpha \frac{A}{d}$$

$$\varepsilon_o = \text{constant of proportionality}$$

$$\varepsilon_{\mathrm{o}} = \text{vacuum permittivity constant}$$

$$\varepsilon_{\mathrm{o}} = 8.85x10^{-12} \frac{C^2}{Nm^2}$$

$$C = \frac{\varepsilon_o A}{d}$$

Capacitor Problems

What is the AREA of a 1F capacitor that has a plate separation of 1 mm?

$$C = \varepsilon_o \frac{A}{D}$$

$$1 = 8.85x10^{-12} \frac{A}{0.001}$$

$A =$ **1.13x10^{8} m^{2}**

$Sides =$ **10629 m**

Is this a practical capacitor to build?

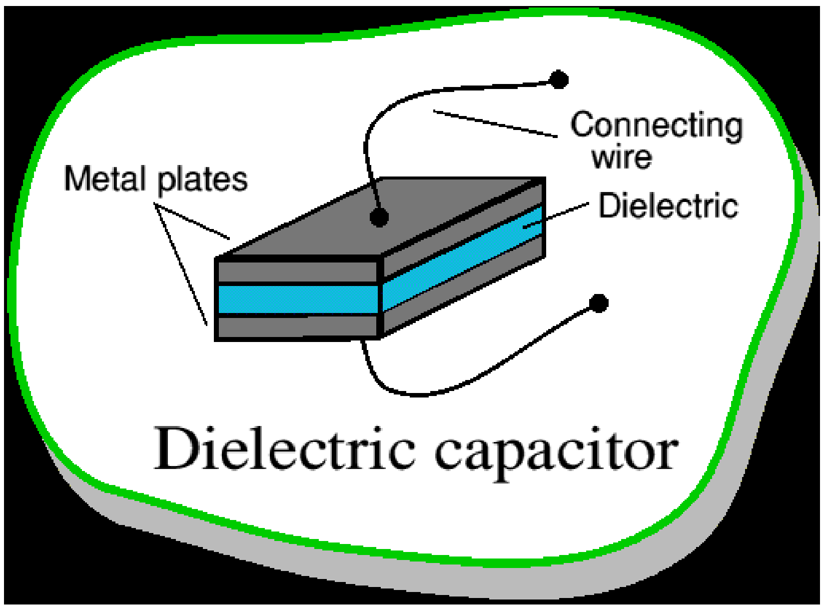
NO! – How can you build this then?

The answer lies in REDUCING the AREA. But you must have a CAPACITANCE of 1 F. **How can you keep the capacitance at 1 F and reduce the Area at the same time?**

Add a DIELECTRIC!!!

Dielectric

Remember, the dielectric is an insulating material placed between the conductors to help store the charge. In the previous example we assumed there was NO dielectric and thus a vacuum between the plates.



$$C = k\varepsilon_o \frac{A}{d}$$

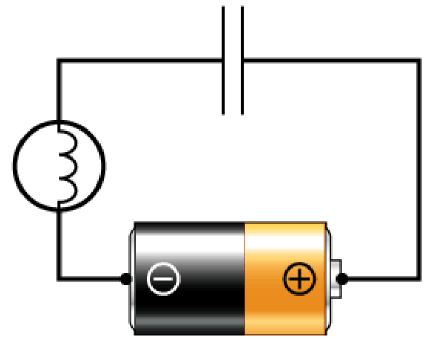
$$k = \text{Dielectric}$$

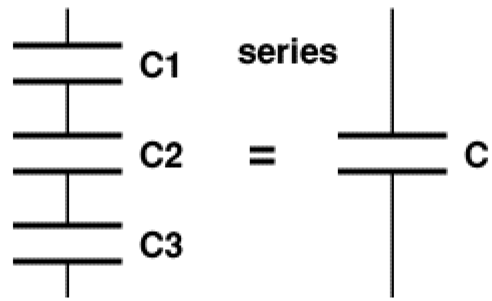
All insulating materials have a dielectric constant associated with it. Here now you can reduce the AREA and use a LARGE dielectric to establish the capacitance at 1 F.

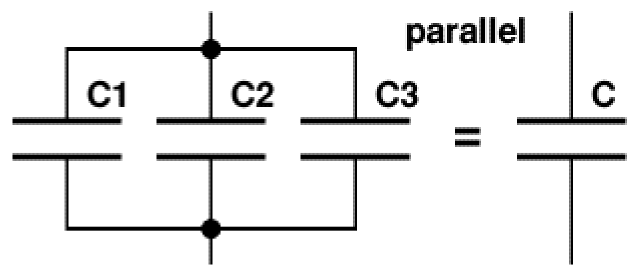
Using MORE than 1 capacitor

Let's say you decide that 1 capacitor will not be enough to build what you need to build. You may need to use more than 1. There are 2 basic ways to assemble them together

- Series – One after another
- Parallel – between a set of junctions and parallel to each other.

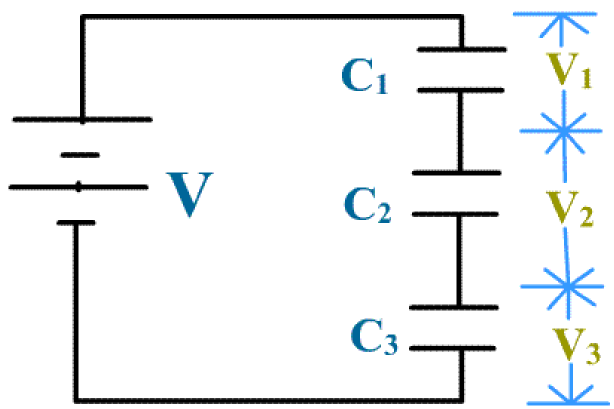






Capacitors in Series

Capacitors in series each charge each other by INDUCTION. So they each have the SAME charge. The electric potential on the other hand is divided up amongst them. In other words, the sum of the individual voltages will equal the total voltage of the battery or power source.



Capacitors in series

$$V_{Total} = V_1 + V_2 + V_3 \ldots$$

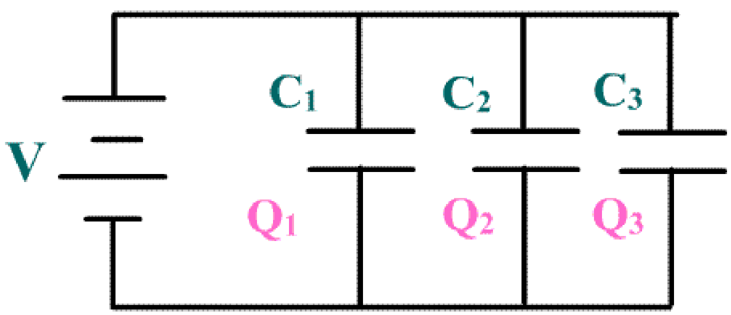
$$V = \frac{Q}{C}$$

$$\frac{Q_T}{C_T} = \frac{Q_1}{C_1} + \frac{Q_2}{C_2} + \frac{Q_3}{C_3}$$

$$\frac{1}{C_{Series}} = \Sigma \frac{1}{C_i}$$

Capacitors in Parallel

In a parallel configuration, the **voltage is the same** because ALL THREE capacitors touch BOTH ends of the battery. As a result, they split up the charge amongst them.



Capacitors in parallel

$$Q_{Total} = Q_1 + Q_2 + Q_3 \ldots$$

$$Q = CV$$

$$C_T V_T = C_1 V_1 + C_2 V_2 + C_3 V_3$$

$$V_T = V_1 = V_2 = V_3$$

$$C_{Parallel} = \Sigma C_i$$

Stored Energy from a Capacitor – A calculus perspective

$$V = \frac{W}{q} = \frac{dW}{dq}, \quad V = \frac{Q}{C}$$

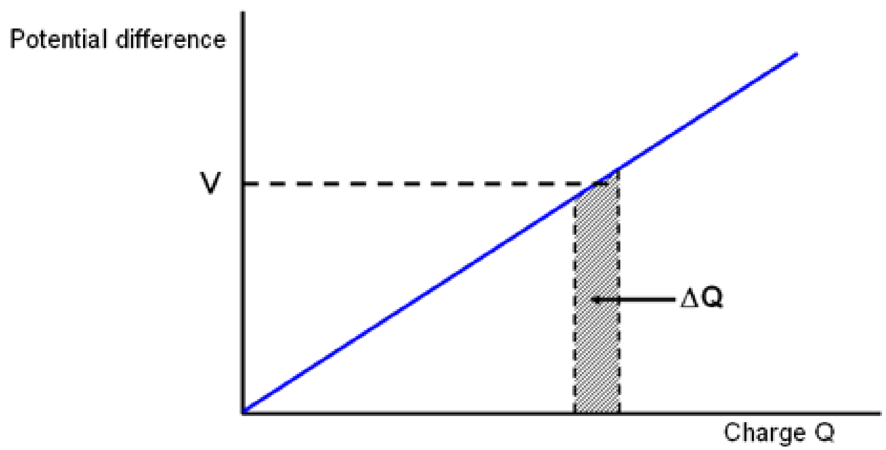
$$dW = \frac{Qdq}{C} = \int_0^Q \frac{Qdq}{C} = \frac{1}{C}\int_0^Q Qdq$$

$$dW = U_c = \left(\frac{1}{C}\right)\left(\frac{Q^2}{2}\right) = \frac{Q^2}{2C}, \quad Q = CV$$

$$U_c = \frac{C^2V^2}{2C} = \frac{1}{2}CV^2$$

Capacitors "STORE" energy

Anytime you have a situation where energy is "STORED" it is called POTENTIAL. In this case we have capacitor potential energy, U_c



Suppose we plot a V vs. Q graph. If we wanted to find the AREA we would MULTIPLY the 2 variables according to the equation for Area.

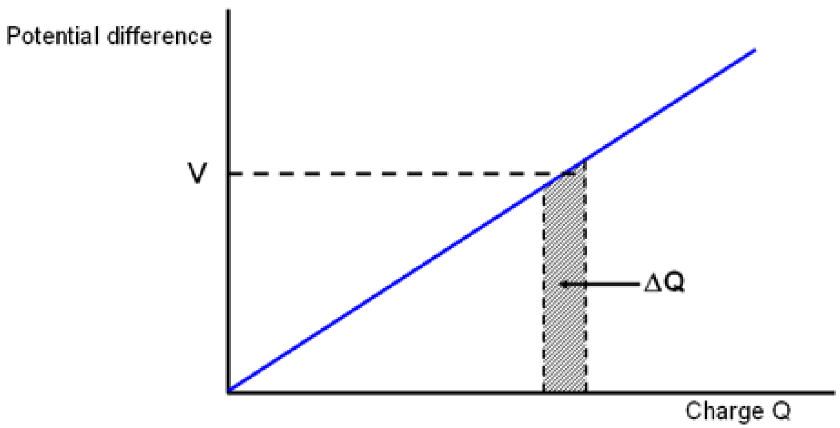
A = bh

When we do this we get Area = VQ

Let's do a unit check!

Voltage = Joules/Coulomb
Charge = Coulombs
Area = **ENERGY**

Potential Energy of a Capacitor



Since the AREA under the line is a triangle, the ENERGY(area) =1/2VQ

$$U_C = \tfrac{1}{2}VQ \qquad C = \frac{Q}{V}$$

$$U_C = \tfrac{1}{2}V(VC) \rightarrow \tfrac{1}{2}CV^2$$

$$U_C = \tfrac{1}{2}\left(\frac{Q}{C}\right)Q \rightarrow \frac{Q^2}{2C}$$

most common form

This energy or area is referred as the **potential energy stored inside a capacitor**.

Note: The slope of the line is the inverse of the capacitance.