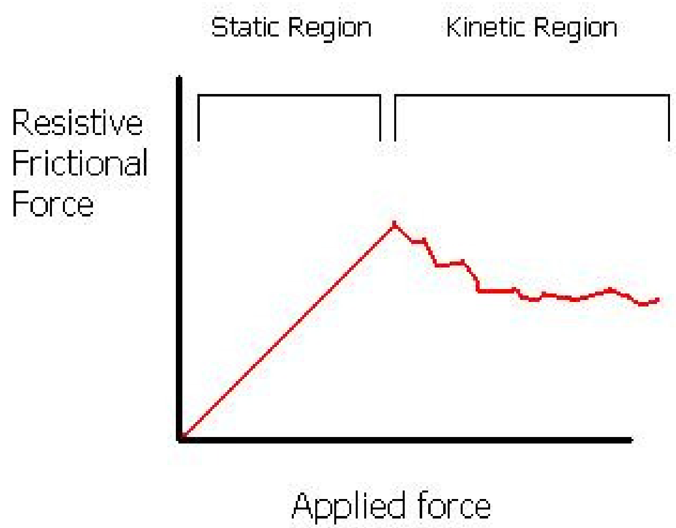
Newton's Laws - continued

Friction, Inclined Planes, N.T.L.

TWO types of Friction

- Static – Friction that keeps an object at rest and prevents it from moving
- Kinetic – Friction that acts during motion



Force of Friction

- The Force of Friction is directly related to the Force Normal.
  - Mostly due to the fact that BOTH are surface forces

$$F_f \;\alpha\; F_N$$

$$\mu = \text{constant of proportionality}$$

$$\mu = \text{coefficient of friction}$$

$$F_{sf} = \mu_s F_N$$

$$F_{kf} = \mu_k F_N$$

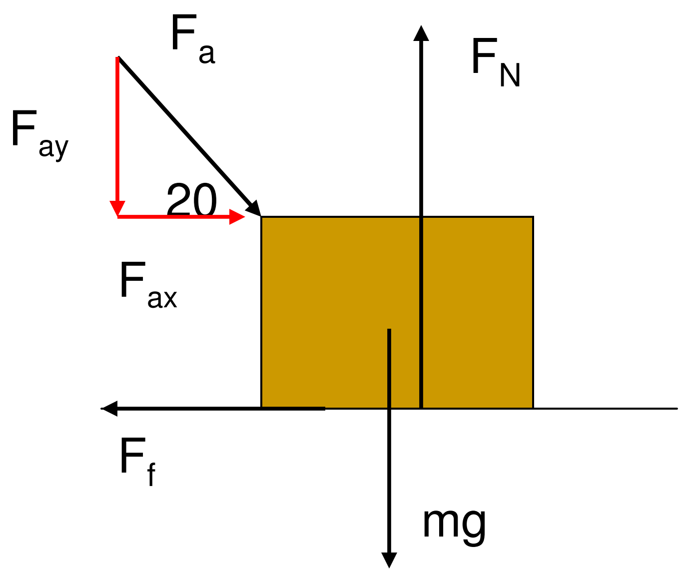
The coefficient of friction is a unitless constant that is specific to the material type and usually less than one.

Note: Friction ONLY depends on the MATERIALS sliding against each other, NOT on surface area.

Example

A 1500 N crate is being pushed across a level floor at a constant speed by a force F of 600 N at an angle of 20° below the horizontal as shown in the figure.

a) What is the coefficient of kinetic friction between the crate and the floor?



$$F_f = \mu_k F_N$$

$$F_f = F_{ax} = F_a \cos\theta = 600(\cos 20) = 563.82N$$

$$F_N = F_{ay} + mg = F_a \sin\theta + 1500$$

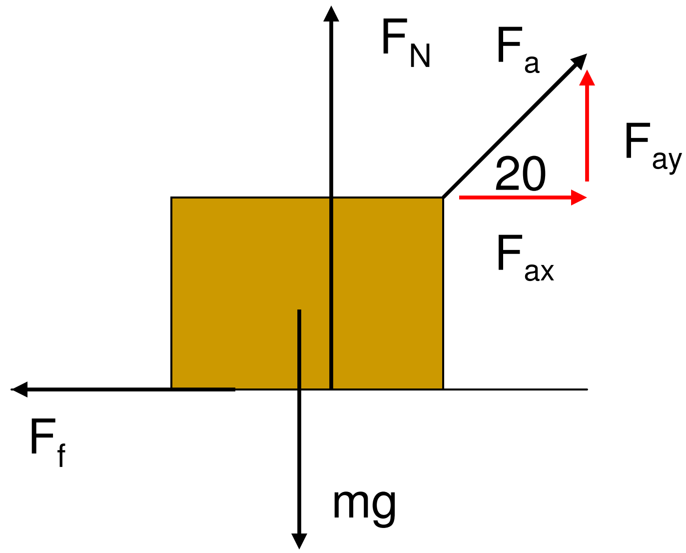
$$F_N = 600(\sin 20) + 1500 = 1705.21N$$

$$563.82 = \mu_k 1705.21$$

$$\mu_k = 0.331$$

Example

If the 600 N force is instead pulling the block at an angle of 20° above the horizontal as shown in the figure, what will be the acceleration of the crate. Assume that the coefficient of friction is the same as found in (a)



$$F_{Net} = ma$$

$$F_{ax} - F_f = ma$$

$$F_a \cos\theta - \mu F_N = ma$$

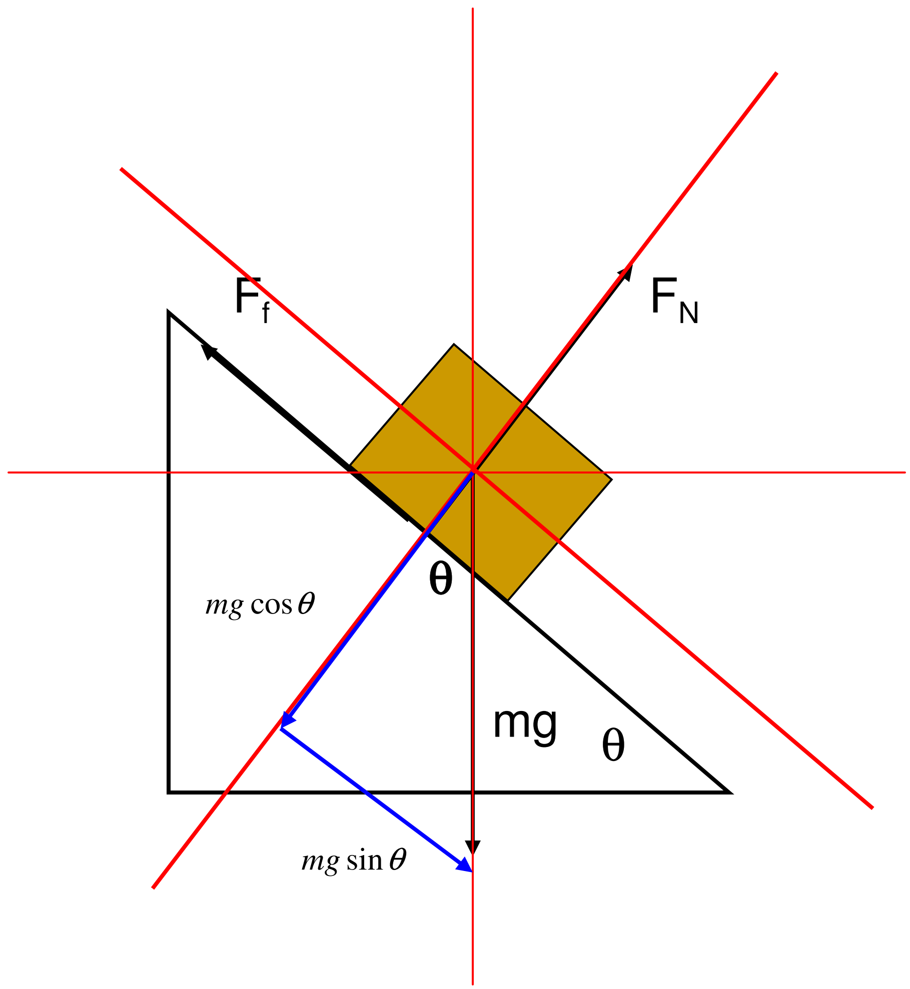
$$F_a \cos\theta - \mu(mg - F_a \sin\theta) = ma$$

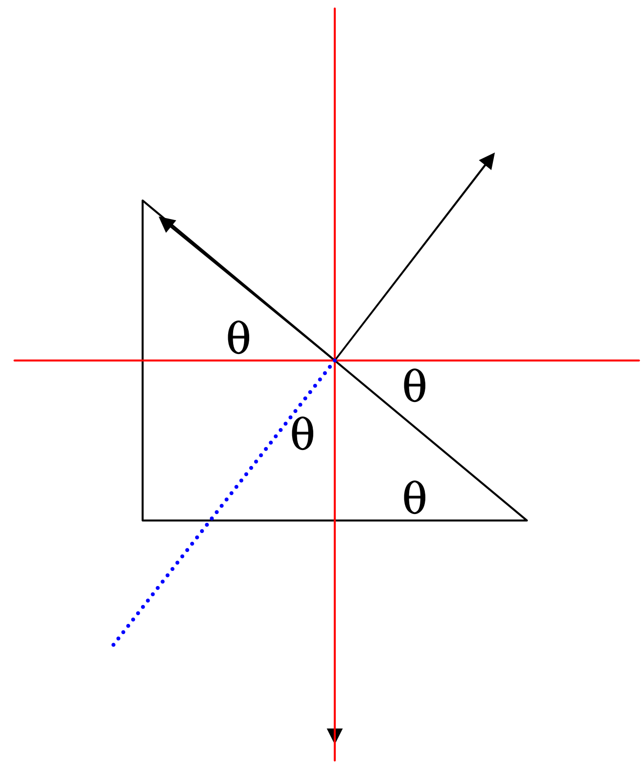
$$600\cos 20 - 0.331(1500 - 600\sin 20) = 153.1a$$

$$563.8 - 428.57 = 153.1a$$

$$a = 0.883\, m/s^2$$

Inclines



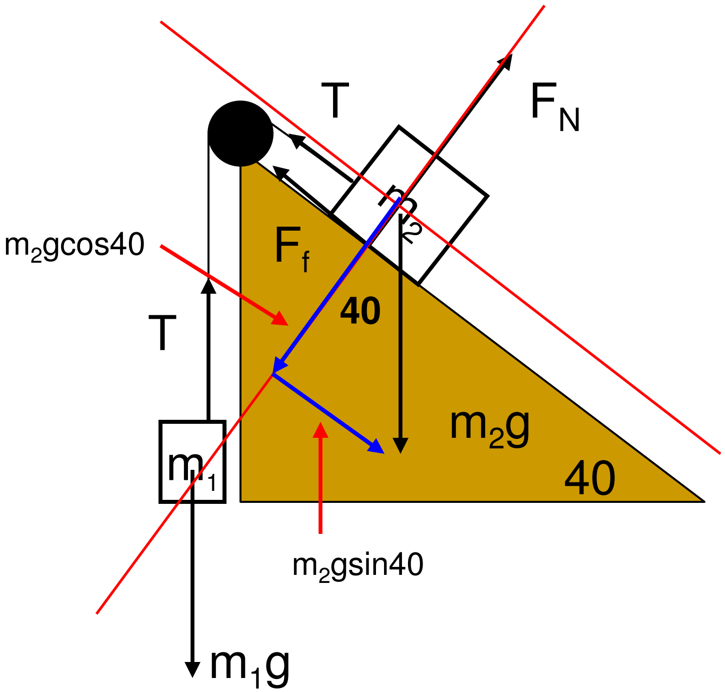


Tips

- Rotate Axis
- Break weight into components
- Write equations of motion or equilibrium
- Solve

Example

Masses m_1 = 4.00 kg and m_2 = 9.00 kg are connected by a light string that passes over a frictionless pulley. As shown in the diagram, m_1 is held at rest on the floor and m_2 rests on a fixed incline of angle 40 degrees. The masses are released from rest, and m_2 slides 1.00 m down the incline in 4 seconds. Determine (a) The acceleration of each mass (b) The coefficient of kinetic friction and (c) the tension in the string.



$$F_{NET} = ma$$

$$T - m_1 g = m_1 a \rightarrow T = m_1 a + m_1 g$$

$$m_2 g \sin\theta - (F_f + T) = m_2 a$$

Example

$$F_{NET} = ma$$

$$T - m_1 g = m_1 a \rightarrow T = m_1 a + m_1 g$$

$$m_2 g \sin\theta - (F_f + T) = m_2 a$$

$$x = v_{ox} t + \tfrac{1}{2} a t^2$$

$$1 = 0 + \tfrac{1}{2} a (4)^2$$

$$a = 0.125\ m/s^2$$

$$T = 4(.125) + 4(9.8) = 39.7\,N$$

$$m_2 g \sin\theta - F_f - T = m_2 a$$

$$m_2 g \sin\theta - F_f - (m_1 a + m_1 g) = m_2 a$$

$$m_2 g \sin\theta - \mu_k F_N - m_1 a - m_1 g = m_2 a$$

$$m_2 g \sin\theta - \mu_k m_2 g \cos\theta - m_1 a - m_1 g = m_2 a$$

$$m_2 g \sin\theta - m_1 a - m_1 g - m_2 a = \mu_k m_2 g \cos\theta$$

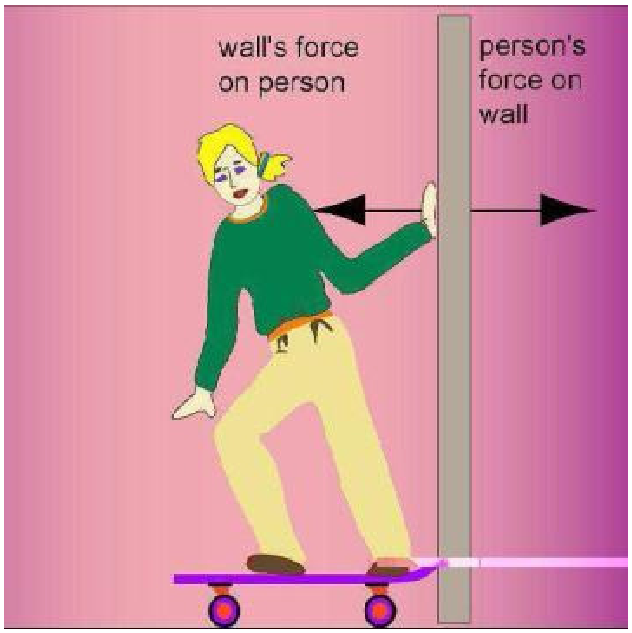
$$\mu_k = \frac{m_2 g \sin\theta - m_1 a - m_1 g - m_2 a}{m_2 g \cos\theta}$$

$$\mu_k = \frac{56.7 - 0.5 - 39.2 - 1.125}{67.57} = 0.235$$

Newton's Third Law

"For every action there is an EQUAL and OPPOSITE reaction.

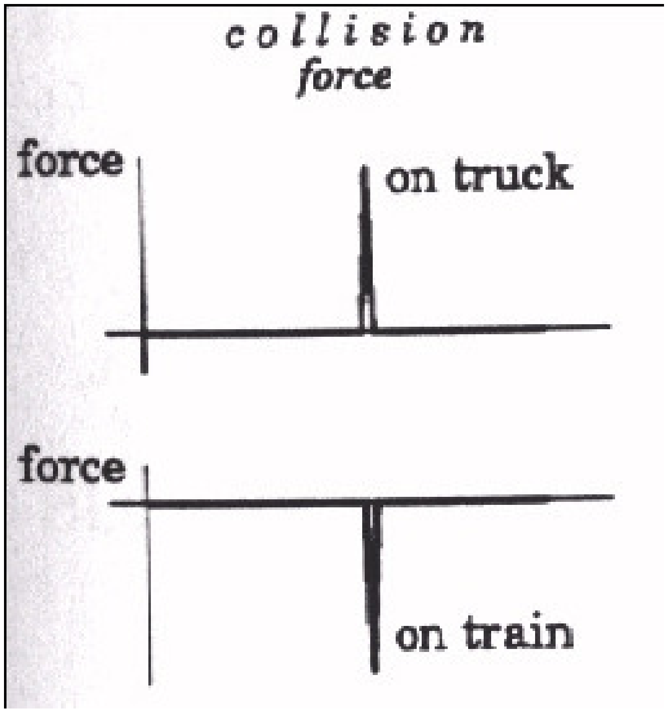
- This law focuses on action/reaction pairs (forces)
- They NEVER cancel out



All you do is SWITCH the wording!

- PERSON on WALL
- WALL on PERSON

N.T.L



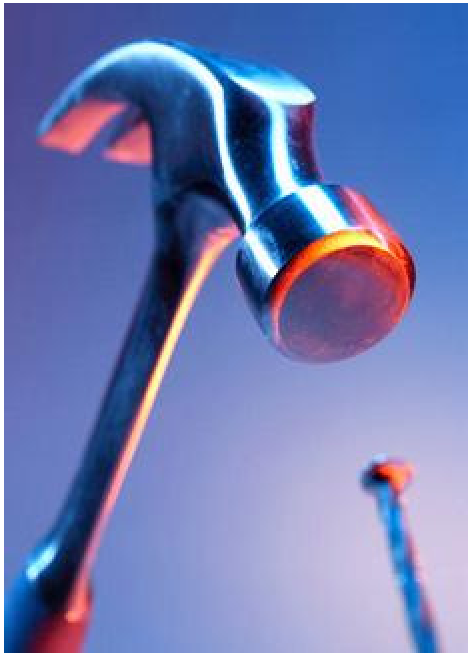
This figure shows the force during a collision between a truck and a train. You can clearly see the forces are EQUAL and OPPOSITE. To help you understand the law better, look at this situation from the point of view of Newton's Second Law.

$$F_{Truck} = F_{Train}$$

$$m_{Truck} A_{Truck} = M_{Train} a_{Train}$$

There is a balance between the mass and acceleration. One object usually has a LARGE MASS and a SMALL ACCELERATION, while the other has a SMALL MASS (comparatively) and a LARGE ACCELERATION.

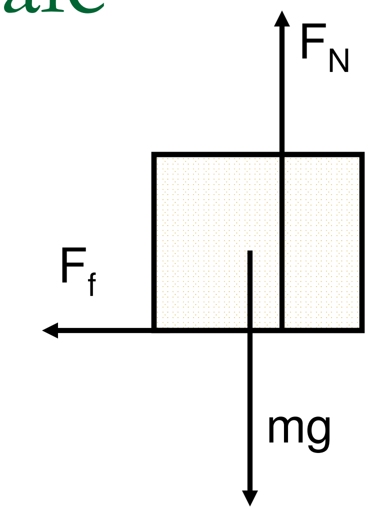
N.T.L Examples



Action: HAMMER HITS NAIL
Reaction: NAIL HITS HAMMER

Action: Earth pulls on YOU
Reaction: YOU pull on the earth

An interesting friction/calc problem…YUCK!



Suppose you had a 30- kg box that is moving at a constant speed until it hits a patch of sticky snow where it experiences a frictional force of 12N.

a) **What is the acceleration of the box?**

b) **What is the coefficient of kinetic friction between the box and the snow?**

$$F_f = \mu_k F_N = \mu_k mg$$

$$12 = \mu_k (30)(9.8)$$

$$\mu_k =$$ **0.04**

$$F_{Net} = ma$$

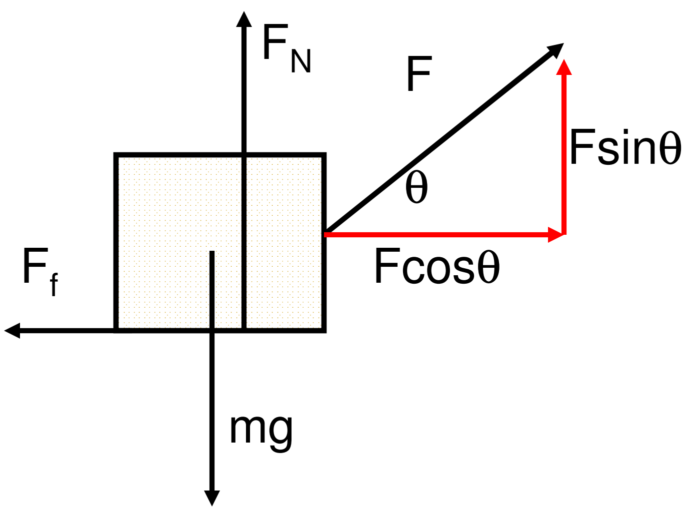
$$F_f = ma \rightarrow 12 = 30a$$

$$a =$$ **0.4 m/s/s**

The "not so much fun" begins….

Now suppose your friend decides to help by pulling the box across the snow using a rope that is at some angle from the horizontal. She begins by experimenting with the ***angle of pull*** and decides that 40 degrees is NOT optimal. At what angle, θ, will the ***minimum force*** be required to pull the sled with a ***constant velocity?***

Let's start by making a function for "F" in terms of "theta" using our equations of motion.



$$F_N + F\sin\theta = mg \rightarrow F_N = mg - F\sin\theta$$

$$F\cos\theta = F_F = \mu_k F_N$$

$$F\cos\theta = \mu_K (mg - F\sin\theta)$$

$$F\cos\theta = \mu_k mg - \mu_k F\sin\theta$$

$$F\cos\theta + \mu_k F\sin\theta = \mu_k mg$$

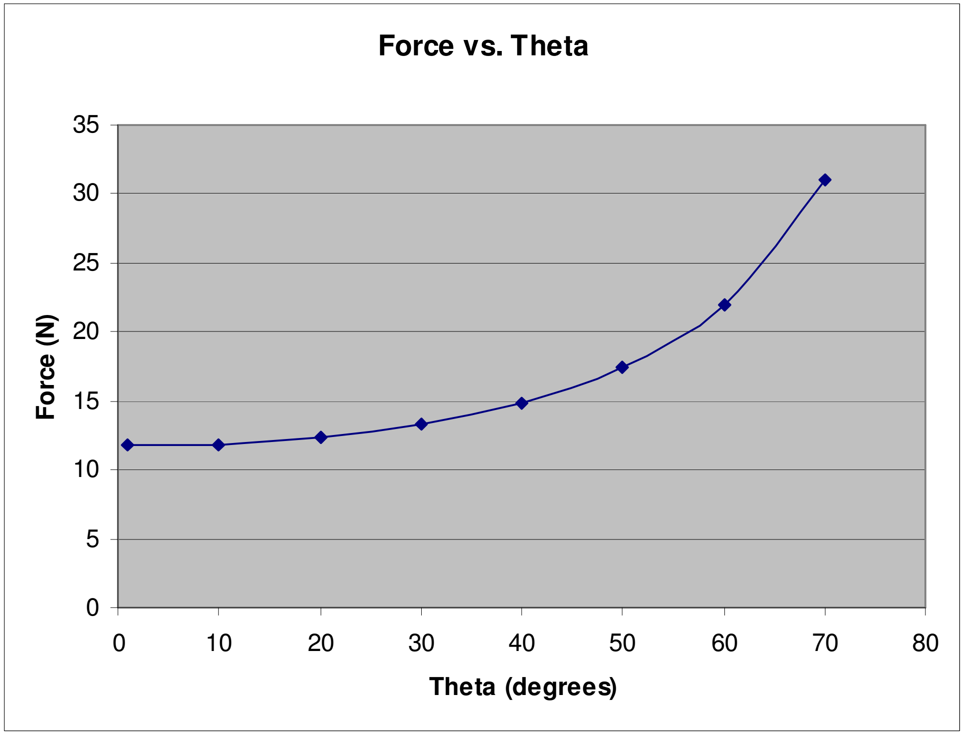
$$F(\cos\theta + \mu_k \sin\theta) = \mu_k mg$$

$$F(\theta) = \frac{\mu_k mg}{\cos\theta + \mu_k \sin\theta}$$

What does this graph look like?

$$F(\theta) = \frac{\mu_k mg}{\cos\theta + \mu_k \sin\theta}$$

Theta	Force
1	11.7536
10	11.8579
20	12.3351
30	13.2728
40	14.8531
50	17.4629
60	21.9961
70	30.9793

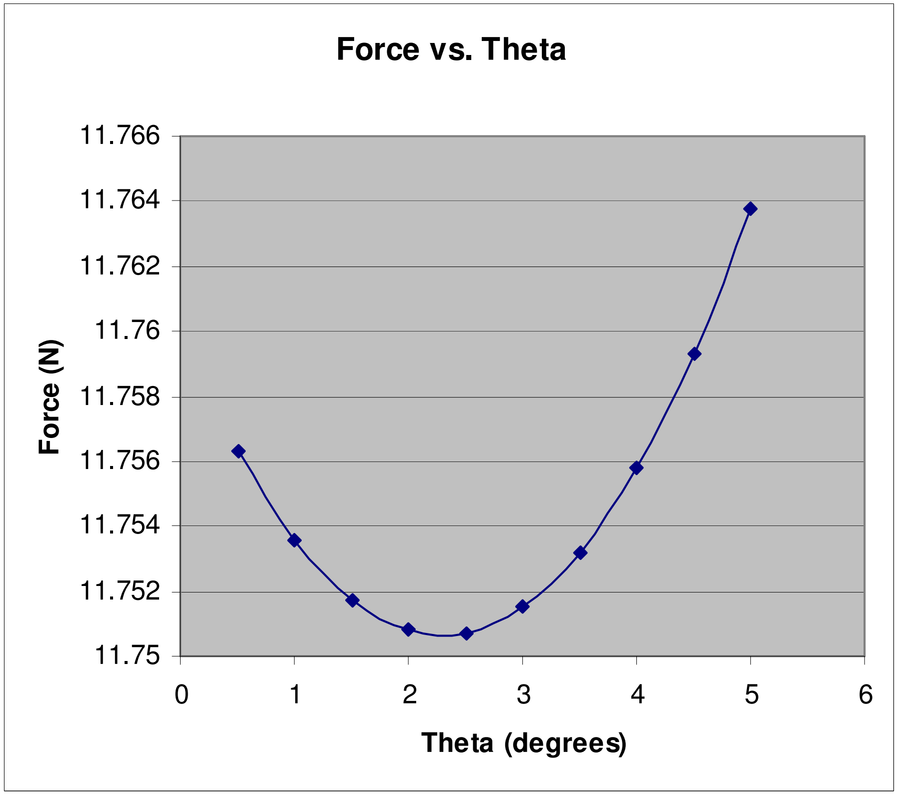


Does this graph tell you the angle needed for a minimum force?

What does this graph look like?

theta	Force
0.5	11.7563
1	11.7536
1.5	11.7517
2	11.7508
2.5	11.7507
3	11.7515
3.5	11.7532
4	11.7558
4.5	11.7593
5	11.7638

$$F(\theta) = \frac{\mu_k mg}{\cos\theta + \mu_k \sin\theta}$$

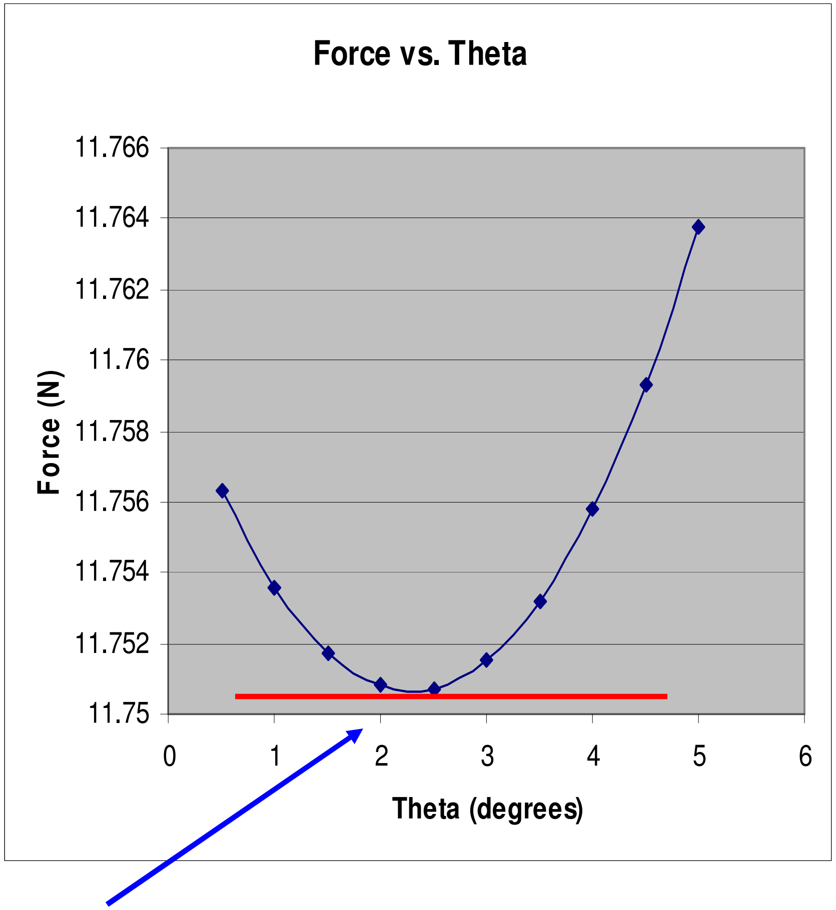


Could this graph tell you the angle needed for a minimum force?

What do you notice about the SLOPE at this minimum force?

Taking the derivative

Here is the point. If we find the derivative of the function and set the derivative equal to ZERO, we can find the ANGLE at this minimum. Remember that the derivative is the SLOPE of the tangent line. The tangent line's slope is zero at the minimum force and thus can be used to determine the angle we need.



This tells us that our minimum force is somewhere between 2 & 3 degrees.

Taking the derivative using the Chain Rule

$$F(\theta) = \frac{\mu_k mg}{\cos\theta + \mu_k \sin\theta}$$

$$\frac{dF}{d\theta} = \frac{d\left(\frac{\mu_k mg}{\cos\theta + \mu_k \sin\theta}\right)}{d\theta} = \frac{d(\mu_k mg(\cos\theta + \mu_k \sin\theta)^{-1})}{d\theta}$$

$$f(x) = \sin(3x^2 + x)$$

$$f'(x) = \underbrace{\cos}_{\text{Derivative of outside function}} \underbrace{(3x^2 + x)}_{\text{Leave inside function alone}} \underbrace{(6x + 1)}_{\text{Derivative of inside function}}$$

Derivative of outside function — **Leave inside function alone** — **Derivative of inside function**

$$f'(x) = (6x + 1)\cos(3x^2 + x)$$

Taking the derivative using the Chain Rule

$$\frac{dF}{d\theta} = \frac{d(\mu_k mg(\cos\theta + \mu_k \sin\theta)^{-1})}{d\theta}$$

$$\mu_k mg = \text{constants}$$

$$\text{-}1(\cos\theta + \mu_k \sin\theta)^{-2} = \text{derivative of outside function}$$

as well as leaving the inside function alone

$$-\sin\theta + \mu_k \cos\theta = \text{derivative of inside function}$$

$$F'(\theta) = \frac{-\mu_k mg(-\sin\theta + \mu_k \cos\theta)}{(\cos\theta + \mu_k \sin\theta)^2}$$

Now we set the derivative equal to ZERO and solve for theta!

Setting the derivative equal to zero

$$F'(\theta) = \frac{-\mu_k mg(-\sin\theta + \mu_k \cos\theta)}{(\cos\theta + \mu_k \sin\theta)^2}$$

$$0 = \frac{-\mu_k mg(-\sin\theta + \mu_k \cos\theta)}{(\cos\theta + \mu_k \sin\theta)^2}$$

$$0 = -\sin\theta + \mu_k \cos\theta$$

$$\sin\theta = \mu_k \cos\theta$$

$$\tan\theta = \mu_k$$

$$\theta = \tan^{-1}(\mu_k) \rightarrow \tan^{-1}(0.04)$$

$$\theta = \mathbf{2.29^\circ}$$