Fluid Dynamics

AP Physics B

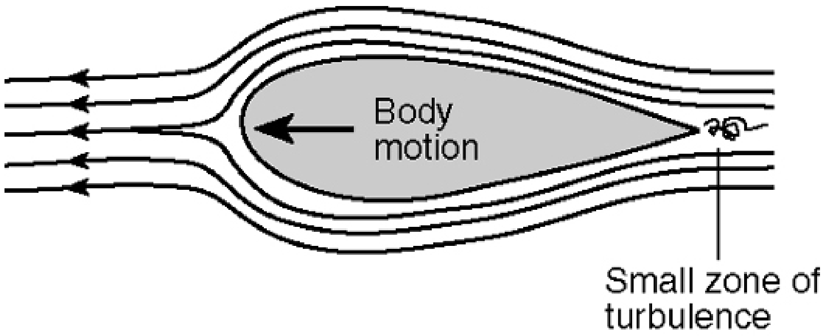
Fluid Flow

Up till now, we have pretty much focused on fluids at rest. Now let's look at fluids in **<u>motion</u>**

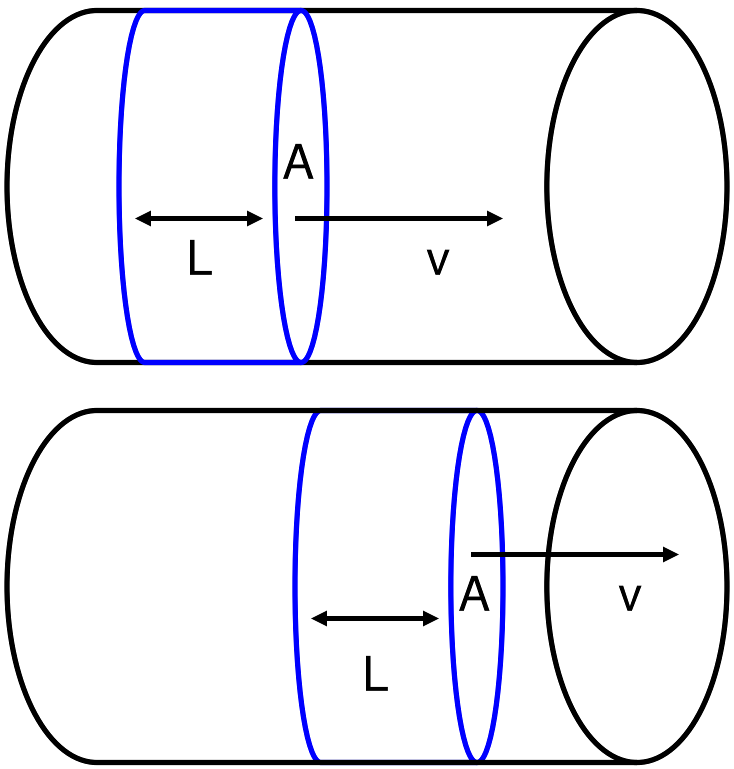
It is important that you understand that an **IDEAL FLUID:**

- **Is non viscous (meaning there is NO internal friction)**
- **Is incompressible (meaning its Density is constant)**
- **Its motion is steady and NON – TURBULENT**

A fluid's motion can be said to be STREAMLINE, or LAMINAR. The path itself is called the streamline. By Laminar, we mean that every particle moves exactly along the smooth path as every particle that follows it. If the fluid DOES NOT have Laminar Flow it has TURBULENT FLOW in which the paths are irregular and called EDDY CURRENTS.



Mass Flow Rate



Consider a pipe with a fluid moving within it.

$$\Delta V = A\Delta l$$

The volume of the blue region is the AREA times the length.

$$\Delta l = v\Delta t$$

$$\Delta V = Av\Delta t$$

Length is velocity times time

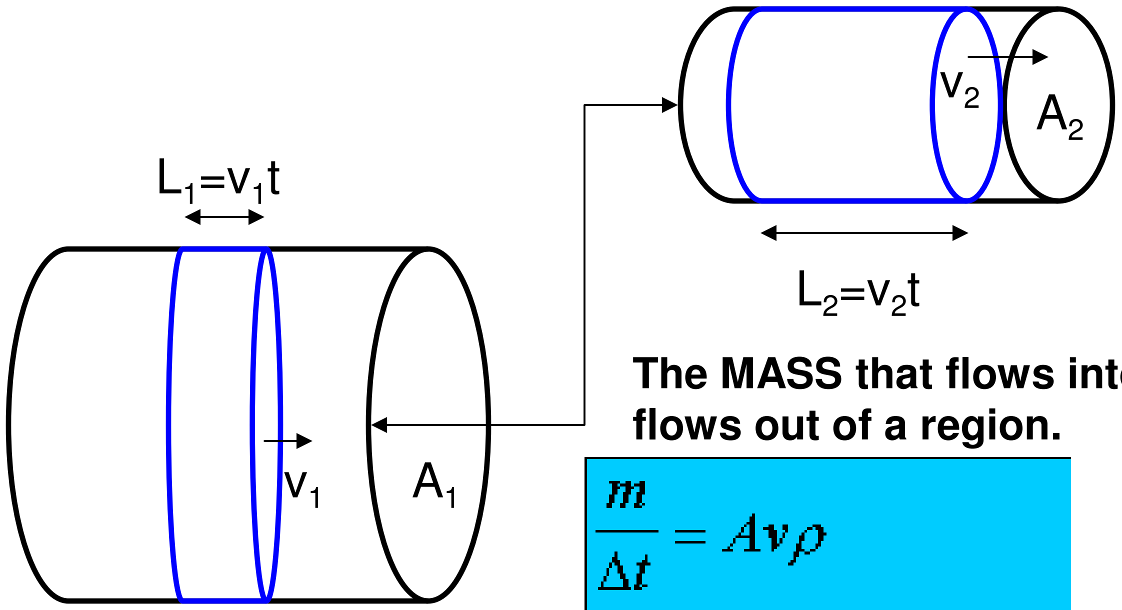
$$\frac{m}{\rho} = Av\Delta t$$

Density is mass per volume

$$\frac{m}{\Delta t} = Av\rho$$

Putting it all together you have MASS FLOW RATE.

What happens if the Area changes?



The first thing you MUST understand is that MASS is NOT CREATED OR DESTROYED!
IT IS CONSERVED.

The MASS that flows into a region = The MASS that flows out of a region.

$$\frac{m}{\Delta t} = Av\rho$$

$$\Delta m_1 = A_1 v_1 \rho_1 \Delta t$$

$$\Delta m_2 = A_2 v_2 \rho_2 \Delta t$$

$$\Delta m_1 = \Delta m_2$$

$$A_1 v_1 \rho_1 \Delta t = A_2 v_2 \rho_2 \Delta t$$

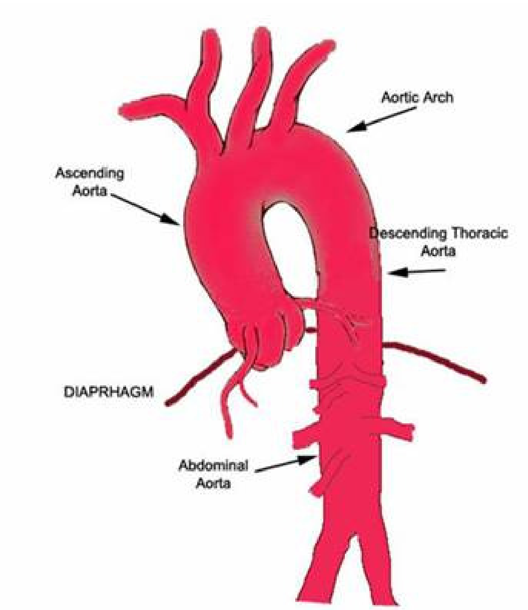
$$A_1 v_1 = A_2 v_2$$

Using the Mass Flow rate equation and the idea that a certain mass of water is constant as it moves to a new pipe section:

We have the Fluid Flow Continuity equation

Example

The speed of blood in the aorta is 50 cm/s and this vessel has a radius of 1.0 cm. If the capillaries have a total cross sectional area of 3000 cm^2, what is the speed of the blood in them?



$$A_1 v_1 = A_2 v_2$$

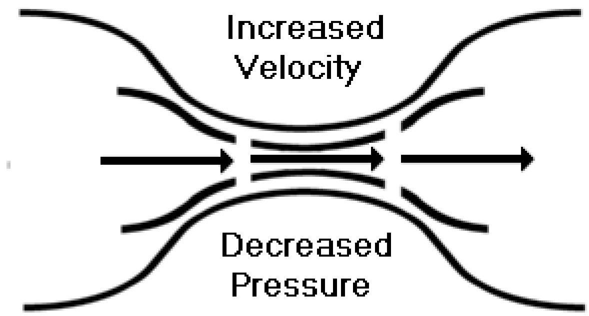
$$\pi r_1^2 v_1 = A_2 v_2$$

$$\pi (1)^2 (50) = (3000) v_2$$

$v_2 =$ **0.052 cm/s**

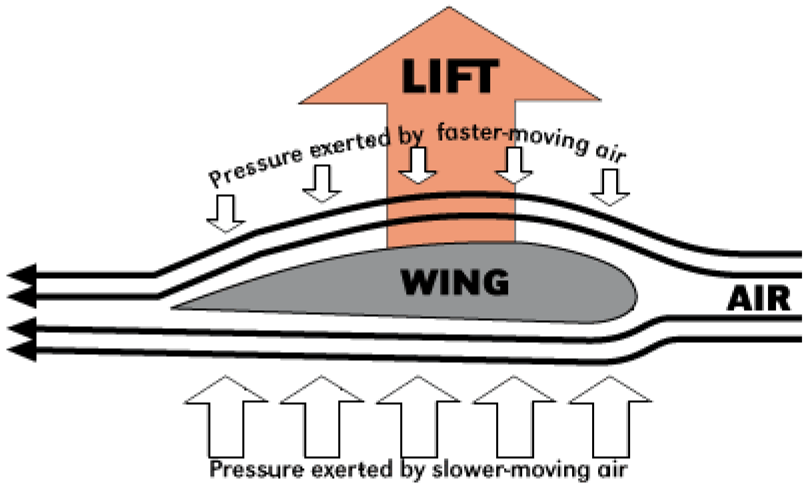
Bernoulli's Principle

The Swiss Physicist Daniel Bernoulli, was interested in how the velocity changes as the fluid moves through a pipe of different area. He especially wanted to incorporate pressure into his idea as well. Conceptually, his principle is stated as: ***" If the velocity of a fluid increases, the pressure decreases and vice versa."***



The velocity can be increased by pushing the air over or through a CONSTRICTION

A change in pressure results in a NET FORCE towards the low pressure region.



Bernoulli's Principle

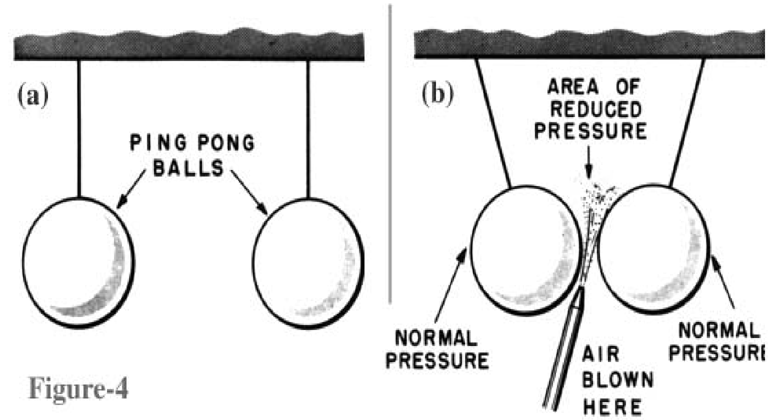
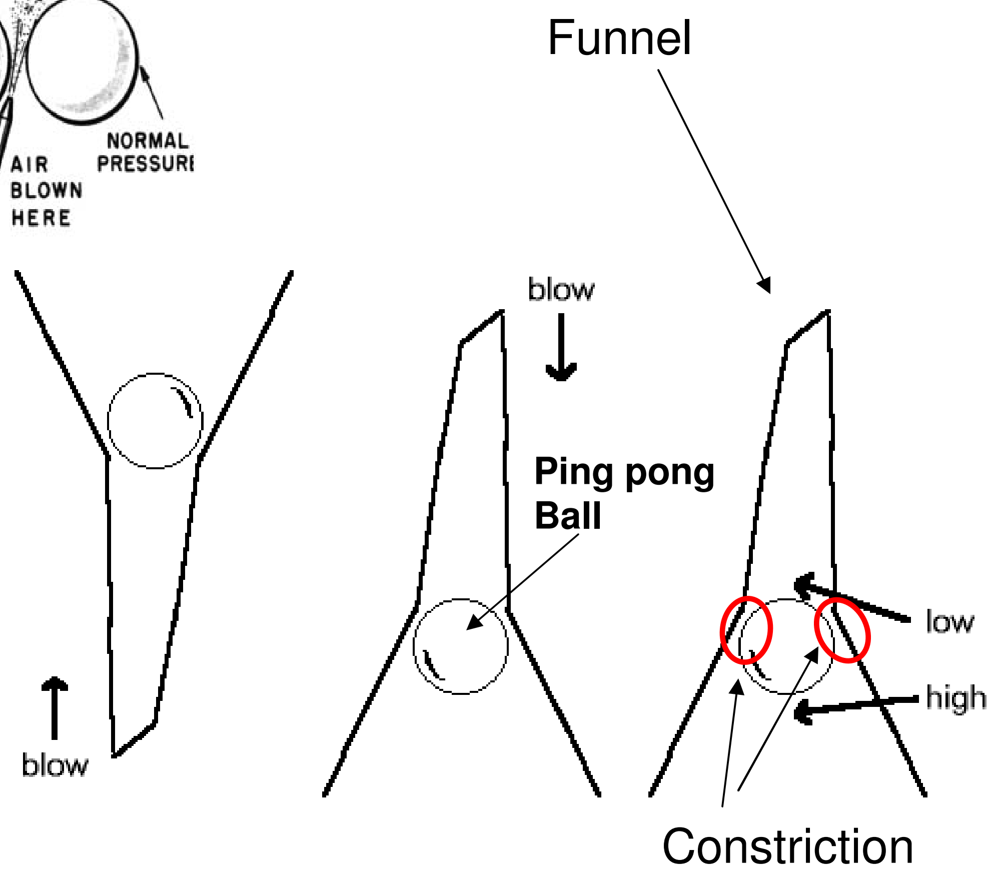
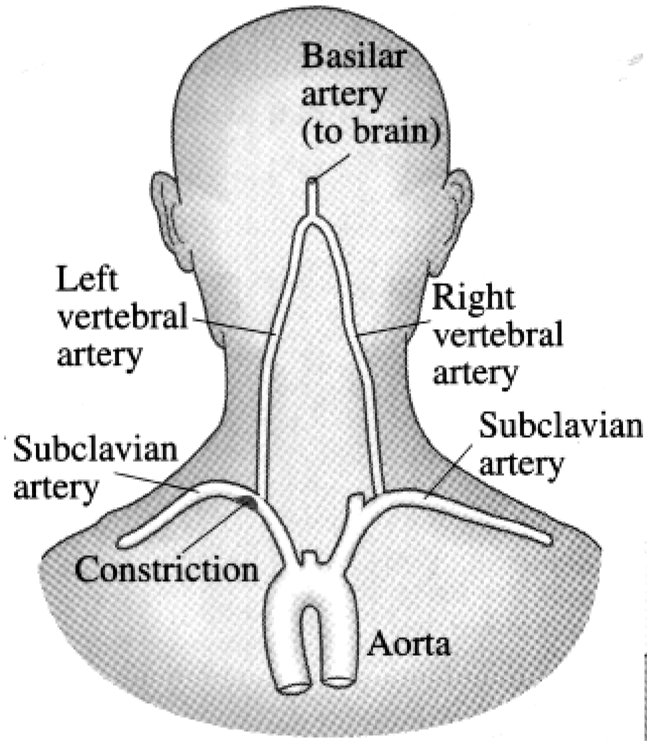


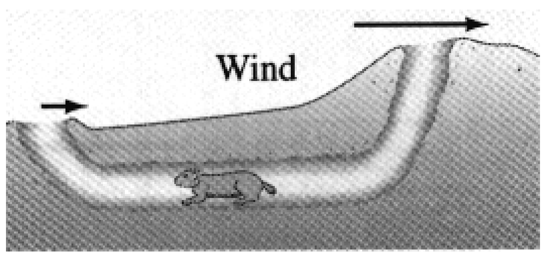
Figure-4



Bernoulli's Principle



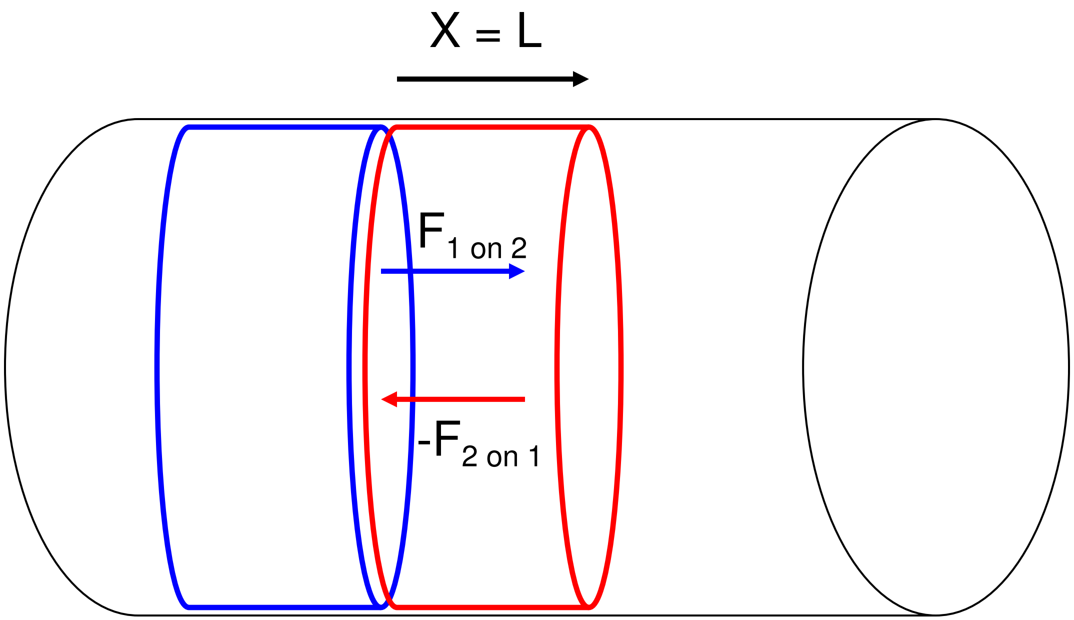
The constriction in the Subclavian artery causes the blood in the region to speed up and thus produces low pressure. The blood moving UP the LVA is then pushed DOWN instead of down causing a lack of blood flow to the brain. This condition is called TIA (transient ischemic attack) or "Subclavian Steal Syndrome.



One end of a gopher hole is higher than the other causing a constriction and low pressure region. Thus the air is constantly sucked out of the higher hole by the wind. The air enters the lower hole providing a sort of air re-circulating system effect to prevent suffocation.

Bernoulli's Equation

Let's look at this principle mathematically.



Work is done by a section of water applying a force on a second section in front of it over a displacement. According to Newton's 3rd law, the second section of water applies an equal and opposite force back on the first. Thus is does negative work as the water still moves FORWARD. Pressure*Area is substituted for Force.

$$W = Fx$$

$$W = F\Delta l$$

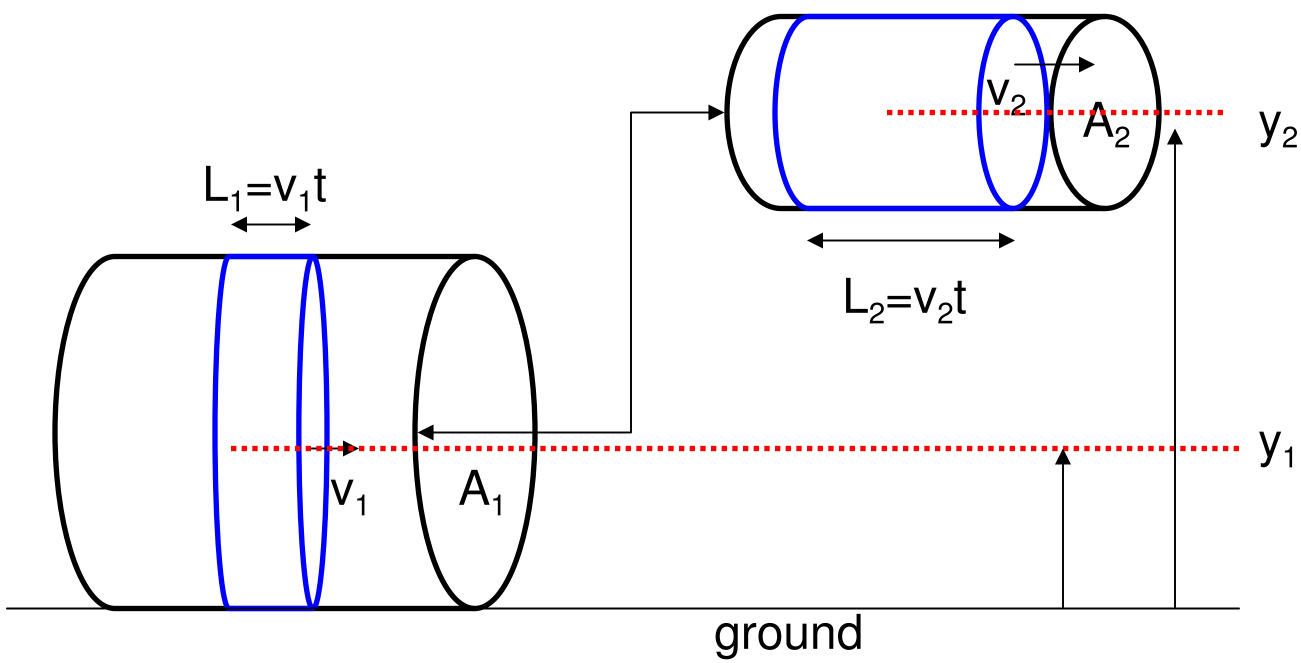
$$W_1 = F_1 \Delta l_1$$

$$-W_2 = F_2 \Delta l_2$$

$$W_1 = P_1 A_1 \Delta l_1$$

$$-W_2 = P_2 A_2 \Delta l_2$$

Bernoulli's Equation



Work is also done by GRAVITY as the water travels a vertical displacement UPWARD. As the water moves UP the force due to gravity is DOWN. So the work is NEGATIVE.

$$-W_3 = mg(y_2 - y_1)$$

Bernoulli's Equation

Now let's find the NET WORK done by gravity and the water acting on itself.

$$W_{net} = W_1 + W_2 + W_3$$

$$W_{net} = P_1 A_1 \Delta l_1 - P_2 A_2 \Delta l_2 - mgy_2 + mgy_1$$

WHAT DOES THE NET WORK EQUAL TO? A CHANGE IN KINETIC ENERGY!

Bernoulli's Equation

$$W_{net} = \Delta KE$$

$$\Delta KE = P_1 A_1 \Delta l_1 - P_2 A_2 \Delta l_2 - mgy_2 + mgy_1$$

$$\frac{1}{2}mv^2 - \frac{1}{2}mv_o^2 = P_1 A_1 \Delta l_1 - P_2 A_2 \Delta l_2 - mgy_2 + mgy_1$$

$$\rho = \frac{m}{V} = \frac{m}{A\Delta l}$$

$$m = \rho A \Delta l$$

Consider that Density = Mass per unit Volume AND that VOLUME is equal to AREA time LENGTH

Bernoulli's Equation

$$\frac{1}{2}\rho A\Delta l v^2 - \frac{1}{2}\rho A\Delta l v_o^2 = P_1 A_1 \Delta l_1 - P_2 A_2 \Delta l_2 - \rho A \Delta l g y_2 + \rho A \Delta l g y_1$$

We can now cancel out the AREA and LENGTH

$$\frac{1}{2}\rho \cancel{A}\cancel{\Delta l} v^2 - \frac{1}{2}\rho \cancel{A}\cancel{\Delta l} v_o^2 = P_1 \cancel{A_1} \cancel{\Delta l_1} - P_2 \cancel{A_2} \cancel{\Delta l_2} - \rho \cancel{A} \cancel{\Delta l} g y_2 + \rho \cancel{A} \cancel{\Delta l} g y_1$$

Leaving:

$$\frac{1}{2}\rho v^2 - \frac{1}{2}\rho v_o^2 = P_1 - P_2 - \rho g y_2 + \rho g y_1$$

Bernoulli's Equation

$$\frac{1}{2}\rho v^2 - \frac{1}{2}\rho v_o^2 = P_1 - P_2 - \rho g y_2 + \rho g y_1$$

Moving everything related to one side results in:

$$P_1 + \frac{1}{2}\rho v_o^2 + \rho g y_o = P_2 + \frac{1}{2}\rho v^2 + \rho g y$$

$$P_1 + \frac{1}{2}\rho v_o^2 + \rho g y_o = constant$$

What this basically shows is that Conservation of Energy holds true within a fluid and that if you add the PRESSURE, the KINETIC ENERGY (in terms of density) and POTENTIAL ENERGY (in terms of density) you get the SAME VALUE anywhere along a streamline.

Example

Water circulates throughout the house in a hot-water heating system. If the water is pumped at a speed of 0.50 m/s through a 4.0 cm diameter pipe in the basement under a pressure of 3.0 atm, what will be the flow speed and pressure in a 2.6 cm-diameter pipe on the second floor 5.0 m above?

1 atm = $1x10^5$ Pa

$$A_1 v_1 = A_1 v_2$$

$$\pi r_1^2 v_1 = \pi r_2^2 v_2$$

$$(0.04)^2 0.50 = (0.026)^2 v_2$$

$v_2 =$ **1.183 m/s**

$$P_o + \frac{1}{2}\rho v_o^2 + \rho g h_o = P + \frac{1}{2}\rho v^2 + \rho g h$$

$$3x10^5 + \frac{1}{2}(1000)(0.50)^2 + (1000)(9.8)(0) = P + \frac{1}{2}(1000)(1.183)^2 + (1000)(9.8)(5)$$

$P =$ **$2.5x10^5$ Pa(N/m^2) or 2.5 atm**