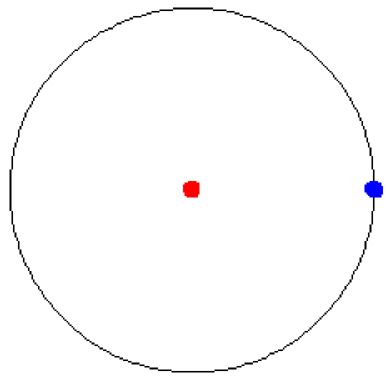
Circular Motion

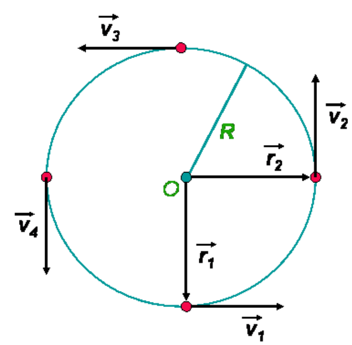
Speed/Velocity in a Circle



Consider an object moving in a circle around a specific origin. The DISTANCE the object covers in ONE REVOLUTION is called the CIRCUMFERENCE. The TIME that it takes to cover this distance is called the PERIOD.

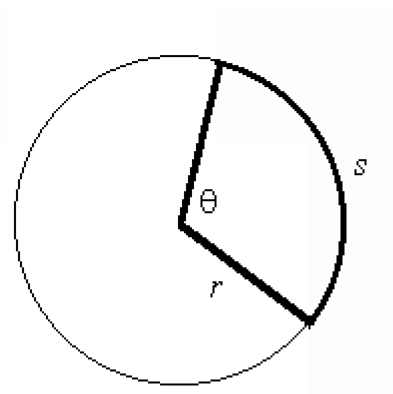
$$\bar{s}_{circle} = \frac{d}{T} = \frac{2\pi r}{T}$$

Speed is the MAGNITUDE of the velocity. And while the speed may be constant, the VELOCITY is NOT. Since velocity is a vector with BOTH magnitude AND direction, we see that the direction o the velocity is ALWAYS changing.



We call this velocity, TANGENTIAL velocity as its direction is draw TANGENT to the circle.

Centripetal Acceleration



Suppose we had a circle with angle, θ, between 2 radaii. You may recall:

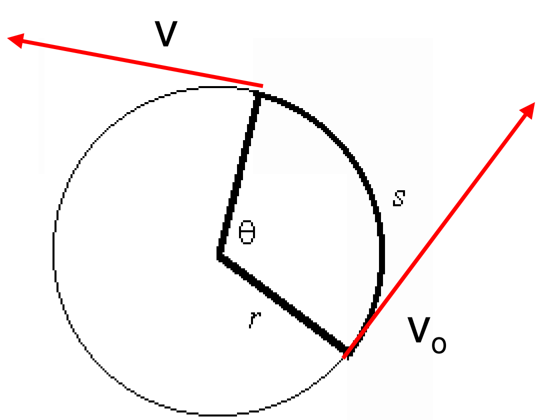
$$\theta = \frac{s}{r}$$

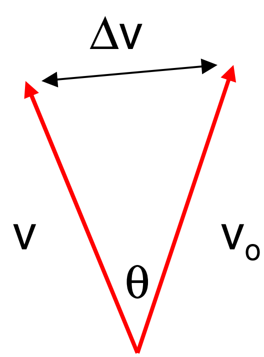
s = arc length in meters

$$\theta = \frac{s}{r} = \frac{\Delta v}{v}$$

$$s = \Delta v t$$

$$\frac{\Delta v t}{r} = \frac{\Delta v}{v}$$



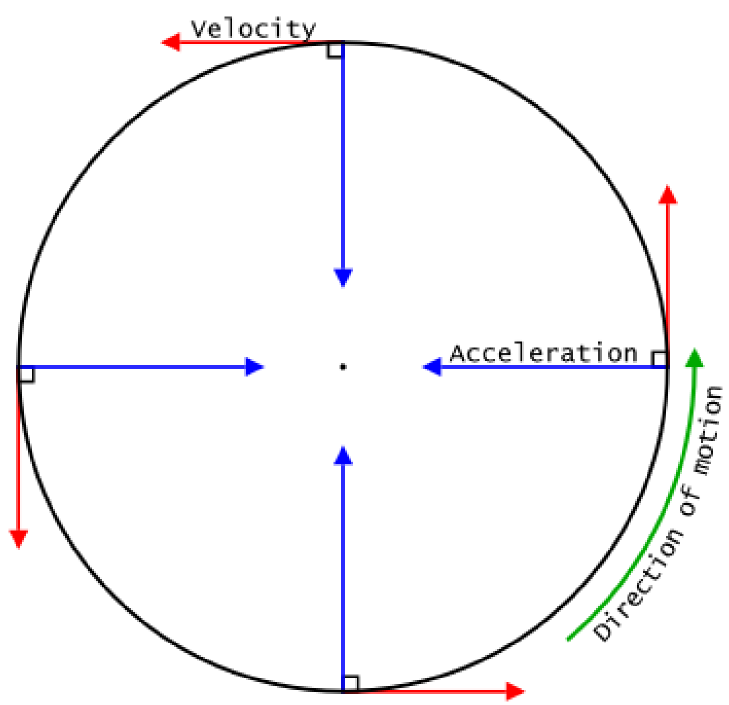


$$\frac{v^2}{r} = \frac{\Delta v}{t} = a_c$$

a_c = *centripetal acceleration*

Centripetal means “center seeking” so that means that the acceleration points towards the CENTER of the circle

Drawing the Directions correctly



So for an object traveling in a counter-clockwise path. The velocity would be drawn TANGENT to the circle and the acceleration would be drawn TOWARDS the CENTER.

To find the MAGNITUDES of each we have:

$$v_c = \frac{2\pi r}{T} \qquad a_c = \frac{v^2}{r}$$

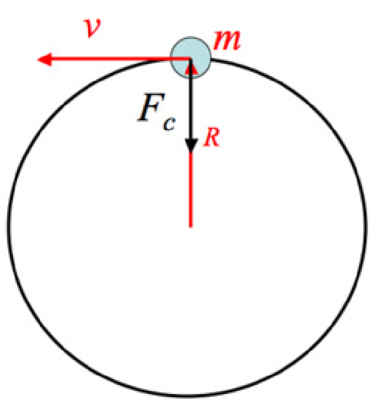
Circular Motion and N.S.L

Recall that according to Newton's Second Law, the acceleration is directly proportional to the Force. If this is true:

$$F_{NET} = ma \quad a_c = \frac{v^2}{r}$$

$$F_{NET} = F_c = \frac{mv^2}{r}$$

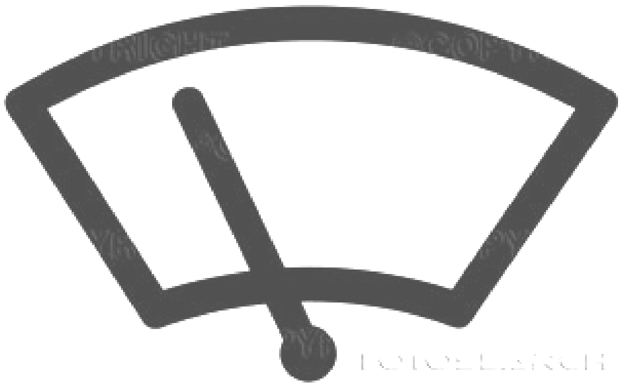
$$F_c = Centripetal\ Force$$



Since the acceleration and the force are directly related, the force must ALSO point towards the center. This is called CENTRIPETAL FORCE.

NOTE: The centripetal force is a NET FORCE. It could be represented by one or more forces. So NEVER draw it in an F.B.D.

Examples



The blade of a windshield wiper moves through an angle of 90 degrees in 0.28 seconds. The tip of the blade moves on the arc of a circle that has a radius of 0.76m. What is the magnitude of the centripetal acceleration of the tip of the blade?

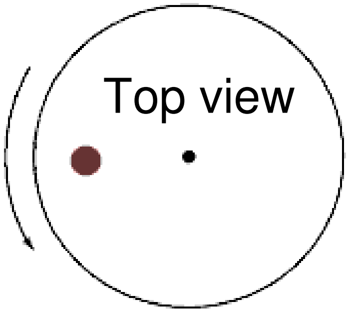
$$v_c = \frac{2\pi r}{T}$$

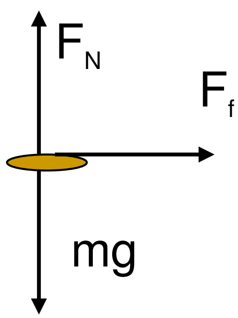
$$v_c = \frac{2\pi(.76)}{(.28 * 4)} = 4.26\, m/s$$

$$a_c = \frac{v^2}{r} = \frac{(4.26)^2}{0.76} = 23.92\, m/s^2$$

Examples

What is the minimum coefficient of static friction necessary to allow a penny to rotate along a 33 1/3 rpm record (diameter= 0.300 m), when the penny is placed at the outer edge of the record?





Side view

$$F_f = F_c$$

$$\mu F_N = \frac{mv^2}{r}$$

$$\mu mg = \frac{mv^2}{r}$$

$$\mu = \frac{v^2}{rg}$$

$$33.3\frac{rev}{\min} * \frac{1\min}{60\sec} = 0.555\, {}^{rev}\!/_{\sec}$$

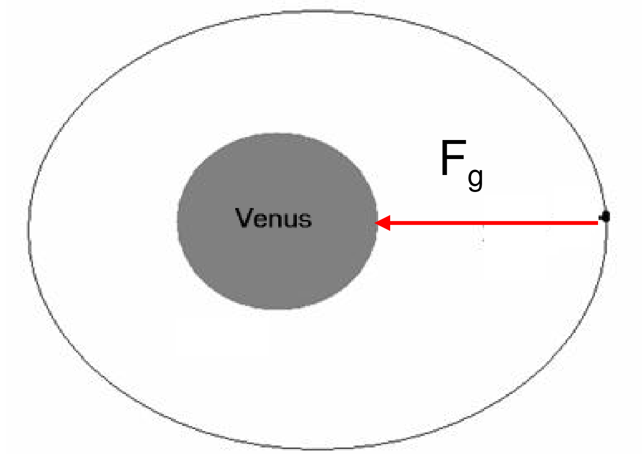
$$\frac{1\sec}{0.555\; rev} = 1.80\, {}^{\sec}\!/_{rev} = T$$

$$v_c = \frac{2\pi r}{T} = \frac{2\pi(0.15)}{1.80} = 0.524\; m/s$$

$$\mu = \frac{v^2}{rg} = \frac{(0.524)^2}{(0.15)(9.8)} = 0.187$$

Examples

Venus rotates slowly about its axis, the period being 243 days. The mass of Venus is 4.87 x 10^{24} kg. Determine the radius for a synchronous satellite in orbit around Venus. (assume circular orbit)



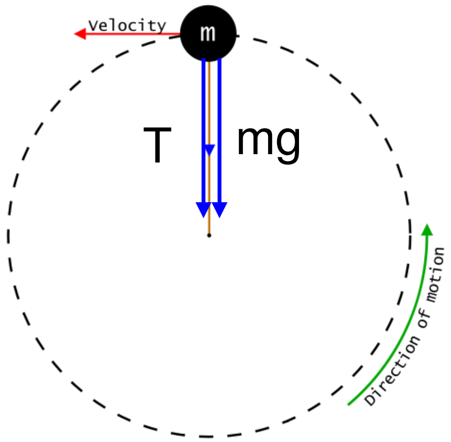
$$F_g = F_c \qquad G\frac{Mm}{r^2} = \frac{mv^2}{r}$$

$$\frac{GM}{r} = v^2 \qquad v_c = \frac{2\pi r}{T}$$

$$\frac{GM}{r} = \frac{4\pi^2 r^2}{T^2} \rightarrow r^3 = \frac{GMT^2}{4\pi^2} \rightarrow r = \sqrt[3]{\frac{GMT^2}{4\pi^2}}$$

$$r = \sqrt[3]{\frac{(6.67x10^{-11})(4.87x10^{24})(2.1x10^7)^2}{4\pi^2}} = \quad 1.54\text{x}10^9 \text{ m}$$

Examples



The maximum tension that a 0.50 m string can tolerate is 14 N. A 0.25-kg ball attached to this string is being whirled in a vertical circle. What is the maximum speed the ball can have (a) the top of the circle, (b)at the bottom of the circle?

$$F_{NET} = F_c = ma_c = \frac{mv^2}{r}$$

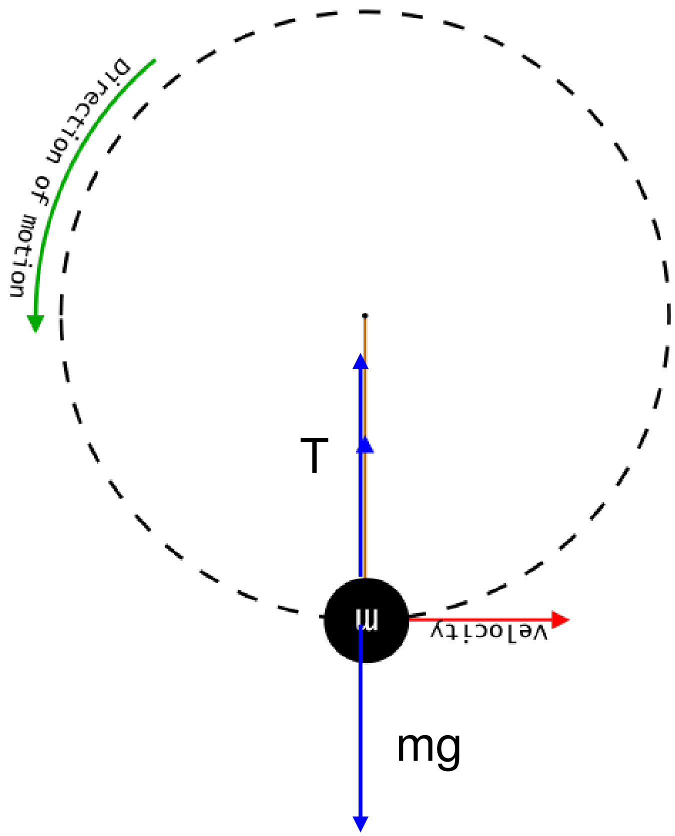
$$T + mg = \frac{mv^2}{r} \rightarrow r(T + mg) = mv^2$$

$$v = \sqrt{\frac{r(T + mg)}{m}} = \sqrt{\frac{0.5(14 + (0.25)(9.8))}{0.25}}$$

$$v = 5.74\, m/s$$

Examples

At the bottom?



$$F_{NET} = F_c = ma_c = \frac{mv^2}{r}$$

$$T - mg = \frac{mv^2}{r} \rightarrow r(T - mg) = mv^2$$

$$v = \sqrt{\frac{r(T - mg)}{m}} = \sqrt{\frac{0.5(14 - (0.25)(9.8))}{0.25}}$$

$$v = 4.81\, m/s$$